Fourier-Mellin Transform in Radiation Therapy

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Overview

• Fourier Transform (FT)
• Log-Polar Transform (LPT)
• Fourier-Mellin Transform (FMT)
• FMT Uses
• Image Registration
• Algorithm Overview
• Image Registration in Radiation Therapy
Fourier Transform

- Jean Baptiste Joseph Fourier, 1807
- Spatial frequency of an image refers to the rate at which the pixel intensities change

original image

FFT
Fourier Transform

- Fourier transform of 2D function:
  \[
  H(u, v) = \iint h(x, y)e^{-i2\pi(ux + vy)} \, dx \, dy
  \]
  \((u, v \text{ – spatial frequencies})\)

- Inverse Fourier transform
  \[
  h(x, y) = \iint H(u, v)e^{i2\pi(ux + vy)} \, du \, dv
  \]

- Shift theorem
  \[
  \mathcal{F}[f(x - a)] = e^{-2\pi iua} F(u)
  \]

- Correlation theorem
  \[
  f(x, y) \circ g(x, y) \Leftrightarrow F^*(u, v)G(u, v)
  \]

- The frequency domain shows the magnitude of different frequency components

- \[
  |H(u, v)| = \sqrt{R^2(u, v) + I^2(u, v)}
  \]

- \[
  \phi(u, v) = \tan^{-1}\left(\frac{I(u, v)}{R(u, v)}\right)
  \]
Log-Polar Transform

- Log-polar transformation can be used to describe both rotation and scaling as a shift.

\[ \log(r_{\text{max}}) - \log(r_0) \]

\[ \phi_{\text{max}} - \phi_0 \]
Fourier-Mellin Transform

- Rotation and scale are converted to shifts along the $\phi$ or log $r$ axis
- If LPT is used on a Fourier image (translation invariant), we get rotation, scale and translation invariance (called Fourier-Mellin transform)
- LPT + FT = FMT
FMT uses

- Object recognition
- Watermarking
- Matching of visual landmarks for robotic homing
- Invariant pattern recognition
- Preprocessing of the fingerprint images
- Image registration
Image Registration

- Several different methods used for image registration:
  - Correlation
  - Transform Methods (including Fourier and Radon transforms)
  - Point Mapping (affine transformation)
  - Edge-based methods
- Most promising registration algorithms are based on FFT
- FMT used for Translation, Rotation, and Scale-Invariant Image Registration
Image Registration in Radiation Therapy

- Portal imaging
- Tomotherapy
- Patient specific IMRT QA
- Etc. (wherever image registration is needed)
Algorithm Overview

• Based on method described by B.S. Reddy and B.N. Chatterji in 1996

• Uses basic approach underlined in paper by H. Xie at al. (implementation of the method)

• Algorithm utilizes properties of the Fourier and log-polar transform
Algorithm Overview

Translation

- Uses correlation and shift theorem of Fourier transform
- Calculates ratio, computes inverse Fourier transform

\[ R(\omega) = \frac{F_2(\omega)F_1^*(\omega)}{|F_2(\omega)| \cdot |F_1^*(\omega)|} \]

- Location of max value gives translation magnitude

Rotation and Scaling

- Basic procedure same as for translation except
- Apply log-polar transformation on FFT before calculating ratio
- Log-polar transformation converts rotation and scaling into translation
- This process is called Fourier-Mellin transform
Final Algorithm

- Written in IDL 5.6 student edition (Research Systems, Inc.)
- FFT applied to plan and plate (film) TIFF images
- Ratio computed
- Location of maximum absolute value found to determine translation
- Translation removed from original plate image
- FFT applied to plan and modified plate images
- Fourier-Mellin transform applied
- Ratio computed
- Location of maximum absolute value found to determine rotation and scale
- Rotation and scaling removed from modified plate image

Plate image is now registered to plan image
IMRT QA case
IMRT QA case
References

• B. Srinivasa Reddy and B. N. Chatterji “An FFT-Based Technique for Translation, Rotation, and Scale-Invariant Image Registration”, IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 5, NO. 8, AUGUST 1996

Thank You!