

CHAPTER 9: THE ROSE MODEL**9.1 Introduction**

In this section, we will discuss a model of human visual perception called the Rose model, named after its formulator Albert Rose. Dr. Rose was a scientist at the Radio Corporation of America (RCA) investigating the basic operating parameters of television in the 1940's and 1950's. In particular, he was trying to relate levels of contrast, resolution, and noise.

Isaac Newton once said, "To see over the horizon, you must stand on the shoulders of giants". This has occurred over and over in science, for example when Johannes Kepler built his model of the heliocentric universe based on the painstaking data of planetary motion collected by Tycho Brahe. In much the same way, Rose built his model of human visual perception on the painstaking data of Richard Blackwell. We therefore will digress and talk about Blackwell's studies, before continuing with the discussion of the Rose model and its application in diagnostic radiology.

Richard Blackwell was a scientist who worked on visual perception studies for the United States Navy during World War II. The Navy was interested in what level of light and how large of an object was required by a sailor to spot an enemy vessel at night. It is obvious that a large light is easier to see than a smaller light, and that a bright light is easier to see than a dim light. But is a large dim light easier to see than a small bright light? The Navy (or someone in the Navy) wanted to know the answer to this question, and provided Blackwell with funds to conduct this research.

The by-gone days of governmental largess for research are apparent in the study that Blackwell performed. For his work, he hired 20 young women and kept them housed and fed in a dormitory built close to his laboratory. For two years he had the women observe simple images of gray circles on plain backgrounds. During each observation, each woman reported whether or not she saw the circle, and in which quadrant of the projection screen it was located. Blackwell and his young female subjects performed thousands of observation studies, and slowly out of this painstaking work emerged a pattern which related the size of the target and the contrast level between the target and background, to the level of illumination (or noise level) of the scene. The results published in graphical form by Blackwell formed the basis for the more theoretical work by Rose.

The theory, outlined by Rose, basically is a probabilistic model of low-contrast threshold detection. Rose's model states that an observer can differentiate two regions of the image, called "target" and "background", only if there is sufficient information to do so. Specifically, if the "signal" is defined to be the difference in the number of photons used to each region, and the "noise" is the statistical uncertainty in each of those regions, the observer needs a certain signal-to-noise ratio to distinguish the target from its background (Figure 9-1). Based on the data of Blackwell, Rose found that this value is in the range of 5 to 7.

We now will derive the Rose equation with a simple statistical model, which assumes that the number of photons used to image the target and the background are Poisson distributed. Furthermore, we will apply this concept to low-contrast situations in which the number of photons in the target is approximately equal to the number of photons in a background region having the same area. Assuming that

$$N = \text{number of photons in target} \sim \text{number of photons in background}$$

- A = area of the target = area of background region
- C = contrast of the signal with respect to the background

then, the contrast (C) is related to the number of detected photons (N), and to the difference (ΔN) in the number of photons in the target and background.

$$C = \Delta N / N \tag{9-1}$$

From this the signal can be expressed in terms of the contrast and the number of detected photons.

$$\text{signal} = \Delta N = C \cdot N \tag{9-2}$$

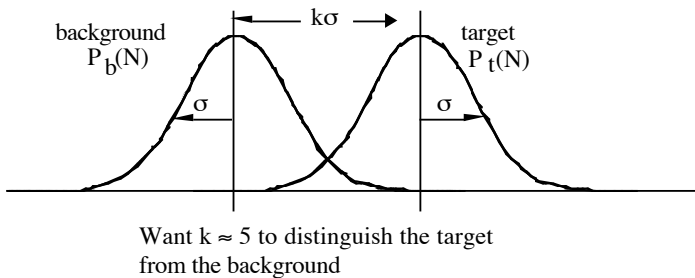
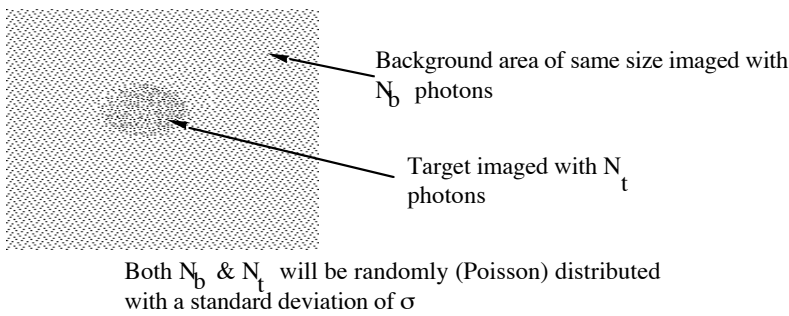
For the very low contrast at the threshold of detectability ΔN approaches zero, so the number of photons in the target and in the background (per unit area) are very close. The noise equals the square root of the number of photons and we will use N from the background region for this.

$$\text{noise} = \sqrt{N} \tag{9-3}$$

so that the signal-to-noise ratio (k) at the threshold of detectability can be expressed as

$$k = \frac{\text{signal}}{\text{noise}} = C\sqrt{N} = C\sqrt{\phi A} \tag{9-4}$$

Probability Distribution of Counts for Threshold Detection in Noise-Limited Images



where ϕ is the photon fluence (e.g., photons per unit area) used to form the image. Equation 9-4 is a mathematical statement of the Rose model.

Rose found that human observers require a signal-to-noise ratio (k) of 5 to 7 to separate a low-contrast target from its background. This value of k therefore is used to establish the threshold of visual detection or perception. This value was derived experimentally based on the work of Blackwell that we discussed earlier.

Figure 9-1. The Rose model specifies that a signal-to-noise ratio of 5 to 7 is needed before the observer is 50% certain of detecting a low-contrast target in a noisy background.

Example 9-1:

Assume that there is an air bubble in a 20 cm thick tank of water (Fig 9-2). Calculate the incident photon fluence and the exposure needed to see an air bubble 1 cm wide as a function of photon energy

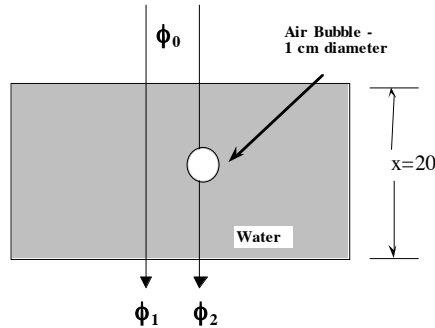


Figure 9-2. A 1 cm diameter air bubble contained in a 20 cm thick volume of water.

E.

Solution:

The Rose model from Eq 9-4 leads to the following for k^2

$$k^2 = C^2 \Phi_1 A \tag{9-5}$$

where

- k = constant with value from 5 to 7
- C = contrast for the object
- Φ_1 = number of photons per unit area to form the image
- A = area of the object we wish to observe

If Φ_0 is the photon fluence in air, then

$$\Phi_1 = \Phi_0 e^{-\mu x} \tag{9-6}$$

is the photon fluence through the water (background region) while

$$\Phi_2 = \Phi_0 e^{-\mu(x-a)} \tag{9-7}$$

is the photon fluence through the water and the air bubble of diameter a. The bubble is imaged with a contrast of

$$C = \frac{\Phi_2 - \Phi_1}{\Phi_1} = \frac{e^{-\mu(x-a)} - e^{-\mu x}}{e^{-\mu x}} = e^{\mu a} - 1 \tag{9-8}$$

The air bubble of diameter a has an area A where

$$A = \frac{\pi a^2}{4} \tag{9-9}$$

From the Rose model, the area A also can be expressed by combining equations 9-5, 9-6, and 9-8, and assuming that k = 5

$$A = \frac{k^2}{C^2 \Phi_1} = \frac{25}{(e^{\mu a} - 1)^2 \Phi_0 e^{-\mu x}} \tag{9-10}$$

and after combining equations 9-9 & 9-10, we see that the incident photon fluence required to see the air bubble is

$$\Phi_0 = \frac{100}{\pi a^2 (e^{\mu a} - 1)^2 e^{-\mu x}} \tag{9-11}$$

where: a = diameter of water bubble
 x = thickness of water tank
 μ = linear attenuation coefficient of water

We can determine an equivalent radiation exposure X corresponding to photon fluence Φ₀ using the following equation from Johns and Cunningham, converted into the appropriate units for this problem

$$= 1.833 \times 10^{-8} \Phi_0 E \left(\frac{\mu_{en}}{\rho} \right)_{air} \left[\frac{gm - mR}{keV} \right] \tag{9-12}$$

Here E is in units of keV, and the mass-energy absorption coefficient of air is in units of cm²/gm. So that the exposure required for the 1 cm diameter air bubble is

$$X = \frac{1.833 \times 10^{-6} E \left(\frac{\mu_{en}}{\rho} \right)_{air} \left[\frac{gm - mR}{keV} \right]}{\pi a^2 (e^{\mu a} - 1)^2 e^{-\mu x}} \tag{9-13}$$

Photon Energy (keV)	$\left(\frac{\mu_{en}}{\rho} \right)_{air}$ (cm ² /gm)	$\left(\frac{\mu}{\rho} \right)_{water}$ (cm ² /gm)	Input Exposure (mR) (1 cm bubble)	Input Exposure (mR) (1 mm bubble)
10	4.533	5.066	1.10E+35	6.1E+41
15	1.242	1.568	3.10E+07	1.6E+12
20	0.4942	0.7613	18.15	3.78E+05
30	0.1395	0.3612	.0177	2.48E+02
40	0.0625	0.2629	.0031	39.49
50	0.0382	0.2245	.0016	19.27
60	0.0289	0.2046	.0012	14.17
80	0.0236	0.1833	.0011	12.58
100	0.0231	0.1706	.0012	13.81
150	0.0249	0.1505	.0017	19.23

For (9-13) the area must be in units of cm^2 . The minimum input exposure required to see 1 cm & 1 mm diameter air bubbles in a 20 cm thick water tank is given for different photon energies. Note that the minimum exposure is seen at about 80 keV for both bubble diameters, but that a much larger exposure is needed to see the smaller bubble.

9.2 Contrast-Detail Curves

A number of investigators, most notably Gerald Cohen, have developed an experimental technique based on the Rose model to evaluate object detectability at the threshold of human visibility in a medical image. This method called contrast-detail curve analysis is based on a graph (called the contrast-detail curve) that relates the threshold contrast to perceive an object in an image as a function of object diameter. A theoretical sketch of a contrast detail curve is shown in the Figure 9.3 and presents our intuitive notion between the relationship of contrast and object size ("detail") in a medical image. Large objects can be visualized at a low contrast while small objects require a high contrast to be visualized. Therefore, if the threshold contrast is displayed on the vertical axis and detail on the horizontal axis, the contrast detail curve starts at the upper left corner (high contrast, small detail) and declines asymptotically toward the lower right corner (low contrast, large detail) in the shape of a hyperbola.

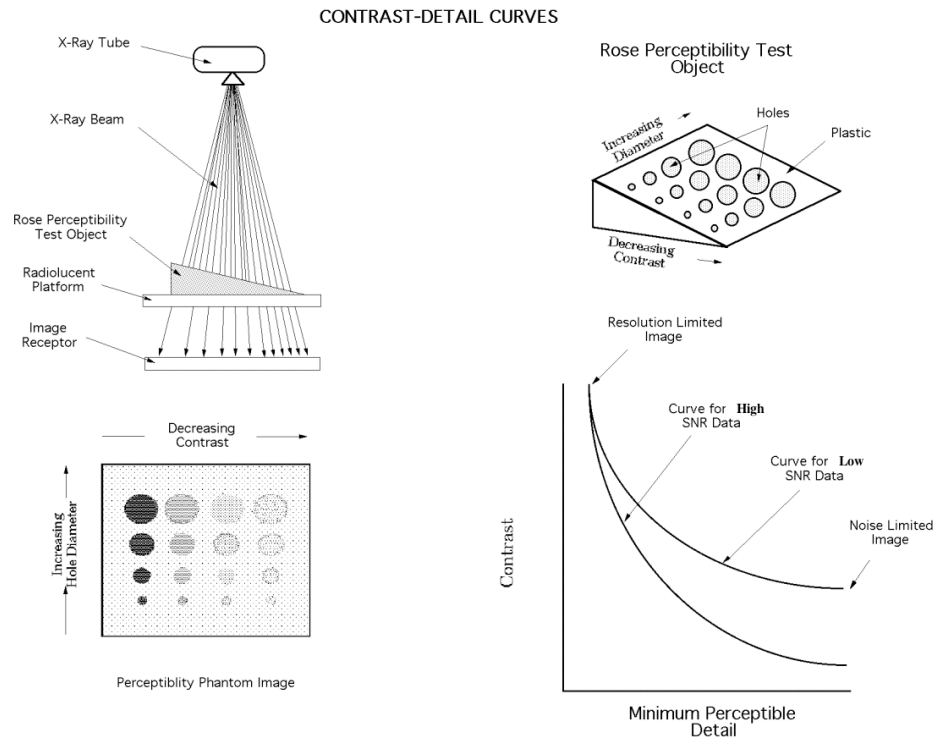


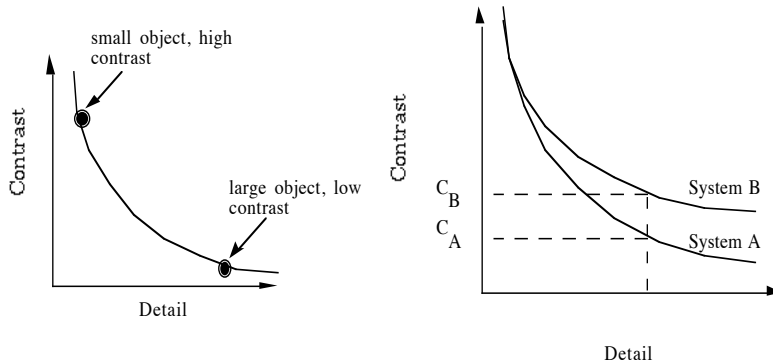
Figure 9-3. Contrast detail curves are obtained by having observers detect circular targets in a radiograph of a Rose model phantom. Resulting contrast detail curve relate indicate the contrast needed to minimally perceive objects of increasing size for high and low x-ray fluences, or high and low SNRs.

System comparisons can be done using two contrast detail curves (Figure 9-4). One curve can be generated from each system, or on the same system under different operating conditions. For example, a contrast detail curve can be generated at one radiographic technique (i.e. kVp, mA, and exposure time), and compared against another curve generated at a different technique. The relative positions of the

contrast detail curves would identify which technique is better at producing images for detecting noise-limited objects at low contrast levels.

9.2.1 Experimental Determination of A Contrast Detail Curve

CONTRAST DETAIL CURVES



An observer can see an object at a lower contrast with System A than with System B, for an object of a given size.

Figure 9-4. Two imaging systems can be compared using contrast detail curves. For a given object size ("detail"), system A can detect the same level of detail at a lower contrast than system B, indicating that system A may be superior to system B.

A contrast detail curve can be generated by having a panel of observers who attempt to detect simple objects in a test image generated with the system under investigation. We will describe how a test object can be designed for film-screen radiography. Of course, the design of an appropriate test object will depend on the system being evaluated, but the general principles will follow those presented in this discussion.

9-5. The phantom is a wedge-shaped piece of plastic (or some other material having a low attenuation) through which holes of various sizes are drilled. The plastic wedge is placed on a film-screen cassette and imaged, and an observer is asked to report holes that can be observed. The plastic thickness provides varying degrees of contrast against which the holes of various diameters (detail) can be observed. A contrast detail curve is generated by graphing contrast vs. hole size for minimally perceptible holes.

A typical test object for contrast detail analysis, called a Rose Model phantom, is shown in Figure

Mathematically, assume we have a piece of plastic of thickness t through which a hole of diameter d is drilled. The plastic is radiographed with the x-ray beam parallel to the axis of the drilled hole. If Φ is the photon fluence (photons/cm²) through the hole, while Φ_1 is the fluence through an adjacent section of plastic, the contrast between the hole and the plastic is

$$C = \frac{\Phi - \Phi_1}{\Phi_1} = e^{\mu t} - 1 \tag{9-14}$$

where μ is the linear attenuation coefficient of the plastic. In 'low contrast' situations (i.e. where $\mu t \ll 1$), the exponential term can be expanded as a Taylor's series $\exp(\mu t) = 1 + \mu t$ so that contrast is approximately proportional to the plastic thickness

$$C \approx \mu t \tag{9-15}$$

The hole is projected onto the radiograph as a circle having diameter **d** (ignoring magnification). Therefore the area of the object we wish to detect is

$$A = \frac{\pi d^2}{4} \quad (9-16)$$

and for low contrast conditions, the photon fluence Φ through the hole is approximately equal to that through the plastic (background). Therefore the Rose model states that

$$k^2 = C^2 \Phi A = (\mu t)^2 \Phi \frac{\pi d^2}{4} \quad (9-17)$$

where k is the signal to noise required by the observer to detect the holes in the radiograph. (For observers under ideal conditions with threshold detection levels k has a value from 5 to 7.)

The Rose phantom is typically constructed with the plastic thickness (i.e. contrast) increasing in one direction and hole diameter (i.e. detail) increasing in the perpendicular direction. Since the contrast and thickness are related by

$$C = \mu t \quad (9-18)$$

and the hole area is related to its diameter by

$$A = \frac{\pi d^2}{4} \quad (9-19)$$

If the hole diameter d is 'increased' by a multiplicative factor while the phantom thickness t is 'decreased' by the same factor, then k^2 in (9-17) remains constant. This means that on the Rose phantom a line connecting holes at increasing diameter with corresponding decreasing contrast have a fixed value of k^2 and represent a line along which the signal-to-noise ratio k is constant for a given photon fluence (Figure 9-5). Hence, a radiograph of the Rose phantom will contain a hypothetical line along which threshold detection occurs for targets of a certain contrast and size. Targets at larger size and contrast on one side of this line can be detected at a higher rate than 50%. Those on the other side of the line with lower contrast and size will be detected with a probability of less than 50%.

DESIGN OF ROSE PHANTOM

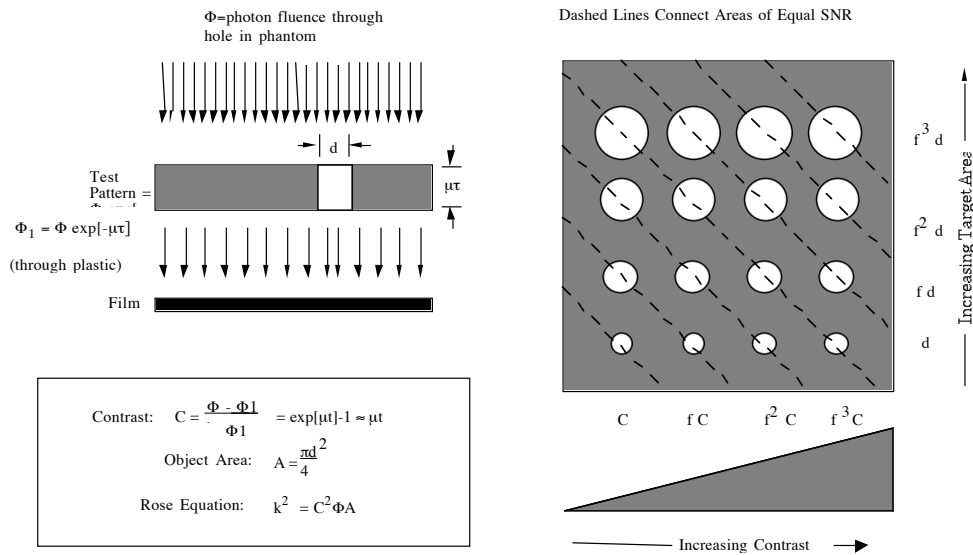


Figure 9-5. In a Rose Model phantom, regions of equal signal-to-noise ratio are obtained by increasing the target diameter and decreasing the material thickness by the same factor (f).

Example 9.2:

Design a Rose phantom to be used in conventional film-screen radiography. The phantom will be radiographed with an exposure of 0.1 μR at an effective energy of 20 keV. Assume that the phantom is made of water-equivalent plastic.

Solution:

Obviously, we have some latitude in the design of the phantom. Let's start by calculating what plastic thickness is required to see a hole with 'diameter' of 2 mm. We know that at energy of 20 keV, since

$$\left(\frac{\mu_{en}}{\rho}\right)_{air} = 0.4941 \frac{cm^2}{gm} \tag{9-20}$$

that the photon fluence for an exposure $X = 0.1 \mu R$ is

$$\Phi = \frac{5.434 \times 10^7 \left[\frac{keV}{gm - mR} \right] \cdot X [mR]}{\left(\frac{\mu_{en}}{\rho}\right) \left[\frac{cm^2}{gm} \right] \cdot E \left[\frac{keV}{photon} \right]} = \frac{551 \text{ photons}}{cm^2} \tag{9-21}$$

From the Rose model and equation 9-17,

$$k^2 = C^2 \Phi A = (\mu t)^2 \Phi \left(\frac{\pi d^2}{4} \right) \quad (9-22)$$

and taking a threshold limit of $k = 5$,

$$t = \frac{2k}{\mu d \sqrt{\pi \Phi}} = 1.58 \text{ cm} \quad (9-23)$$

Where $\mu = 0.7613 \text{ cm}^{-1}$ is the linear attenuation coefficient of the water equivalent phantom material at 20 keV.

We have now established a single thickness and hole diameter for the phantom. From our preceding discussion, we know we can increase the phantom thickness by a multiplicative factor while we decrease hole diameter in the opposite direction by the same factor. Experience has shown that this factor should be no larger than $\sqrt{2}$. Using this factor, we will select hole diameters of

1 mm, 1.41 mm, 2 mm, 2.83 mm 4 mm

and phantom thickness of

0.79 cm, 1.12 cm, 1.58 cm, 2.23 cm, 3.16 cm

both of which change by a factor of $\sqrt{2}$ from their adjacent values.

CHAPTER 9: HOMEWORK PROBLEMS

1. A patient is suspected of having a tumor located in his lung. Assume that the body thickness in the area of the lung has a thickness equivalent to 10 cm of water and that the soft tissue has an attenuation coefficient (after the patient) corresponding to an HVL of 3 cm H₂O.
 - (a) Calculate the entrance exposure required to detect a 1 cm tumor (water equivalent) sitting in the lung. For simplicity assume relative path lengths of 10 and 11 cm rather than worrying about the replacement of 1 cm of low-density lung tissue.
 - (b) Repeat the calculation for a 1 mm tumor.
 - (c) Repeat (b) assuming path lengths of 15.0 and 15.1 cm (i.e. a 15 cm thick body region) if scatter is neglected.

2. Compare the x-ray exposure required for noise-limited radiographic imaging for a small cubic object of dimension "a" and for a cube of similar material but dimension "2a".

3. For a fixed radiographic entrance exposure, how is the signal-to-noise ratio of an image affected if the diameter of the object is halved and its contrast tripled.

4. We wish to determine the incident exposure required to image a 1 mm diameter, 1 mm long arterial stenosis that is located in 20 cm of water-equivalent tissue. Assume that the artery contains 10 mg/cm² of iodine at a density of 2 gm/cm³. Calculate the incident exposure for the energies listed in the table below. Using this information, determine the optimal energy for imaging the stenosis.

TABLE OF ATTENUATION COEFFICIENTS FOR PROBLEM 4
 All mass attenuation coefficients given in units of cm²/gm.

Photon Energy (keV)	$\left(\frac{\mu_{en}}{\rho}\right)_{air}$	$\left(\frac{\mu}{\rho}\right)_{water}$	$\left(\frac{\mu}{\rho}\right)_{Iodine}$
10	4.533	5.066	162.7460
15	1.242	1.568	54.5977
20	0.4942	0.7613	25.0943
30	0.1395	0.3612	8.4212
40	0.0625	0.2629	22.3827
50	0.0382	0.2245	12.5016
60	0.0289	0.2046	7.6938
80	0.0236	0.1833	3.5496
100	0.0231	0.1706	1.9562
150	0.0249	0.1505	0.6984

5. The Rose model phantom given in the example in Example 9-2 is used in various imaging situations.
- (a) Draw a contrast-detail curve for this phantom at the imaging technique (0.1 μR per exposure, 20 keV effective energy) given in example 9-2.
 - (b) Draw in the contrast-detail curve if the exposure time is doubled while the exposure rate is maintained at the same level given in (a).
 - (c) If the effective energy of the beam is changed from 20 keV to 30 keV, but the exposure is still 0.1 μR , how does this change the contrast detail curve? Answer this question quantitatively.
 - (d) If the thickness of the phantom is doubled how does this change the contrast detail curve obtained in (a).
 - (e) The Rose phantom is used with a fluorographic system where the electronic noise equals the quantum statistical noise. How does this change the contrast detail curve obtained in (a)?
 - (f) The Rose phantom is used in a situation where the scatter fraction is 75%? How does this change the contrast-detail curve from that given in (a)?