**Assessing Spatial Resolution**. When simpler methods do not provide sufficient accuracy, model function fitting of spread functions can be used estimate spatial resolution. One approach is to model the point spread function PSF(x,y) or line spread function LSF(x) using normalized Gaussian functions. The Gaussian model is considered adequate for FWHM estimates when the overall spread function is the result of several component spread functions. This is justified in part since multiple convolutions can lead to a net system spread function that is approximately Gaussian, a result predicted by the Central Limit Theorem.

A simple two dimensional (2-D) PSF Gaussian model is

$$PSF(x,y) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2}\left(\frac{x-x_c}{\sigma_x}\right)^2} \cdot \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{1}{2}\left(\frac{y-y_c}{\sigma_y}\right)^2} = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2}\left[\left(\frac{x-x_c}{\sigma_x}\right)^2 + \left(\frac{y-y_c}{\sigma_y}\right)^2\right]}. \qquad \text{6-1S}$$

This model is correct when a PSF is fully characterized by spread characteristics in the x and y directions and the measured PSF is aligned with these axes. For this case the 2-D PSF is just the product of two orthogonal 1-D PSFs. The one dimensional Gaussian model for LSFs is

$$LSF(x,y) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{1}{2}\left(\frac{x-x_c}{\sigma_x}\right)^2}. \qquad \text{6-2S}$$

In these models $x_c$ and $y_c$ are the centers of the Gaussians and $\sigma_x$ and $\sigma_y$ are the corresponding standard deviations. We can estimate the index of spatial resolution “FWHM” from standard deviations as

$$FWHM = 2\sqrt{2\ln(2)}\sigma \approx 2.35\sigma. \qquad \text{6-3S}$$

<u>Acquiring Spread Function Data</u>. There are numerous publications and sources of phantoms and tools for the assessment of system PSF or LSF for medical imagers. It is best to follow guidelines given by organizations such as NEMA or AAPM to ensure that your results will be comparable with measures made by others. Align major axes of the spread functions with appropriate x- and y-axes in images. If images are acquired digitally use the smallest sample (pixel and/or voxel) spacing available. For model fitting purposes the spread function data samples should span a range about the center of approximately +/-6-7 times the anticipated FWHM. At these distances the spread function value drops to ~1% of its peak value. You should try to achieve a signal-to-noise ratio (SNR) of ~ 10:1 at the extreme points. The noise can be estimated at distances 25-30X the anticipated FWHM from the spread function. You should acquire test images to determine the best imaging parameters to ensure adequate SNR.

<u>Preprocessing Spread Function Data</u>. You may need to correct for non-uniform and/or nonlinear response of the imaging system. Uniformity can be evaluated in a broad field of view image and correction applied to adjust uniformity to the desired level. If the imaging system has nonlinear spatial and/or signal response these must be corrected. While local signal nonlinearity is small in many imaging systems, those that incorporate a nonlinear device (such as film) will require correction. For example, calibration data for a film (or from known H&D curve) should be used to convert film density as measured by a scanning densitometer to relative exposure values.

Since both PSF(x,y) and LSF(x) model functions are normalized spread functions, measured spread function must be normalized. Dividing each sampled value by the sum of all sample values readily does this.

Fitting Model Functions to Data. A variety of fitting methods are available in math software applications such as Matlab, Mathematica, and MathCad. Fitting involves adjusting model function parameters to minimize a cost function, usually the mean square error (MSE) between model function and sample data. General MSE methods require starting or seed values for model parameters. Seed values can be estimated from the data, i.e. $x_c$, and $y_c$ can be estimated as the x- and y-centroids. Seed values for $\sigma_x$ and $\sigma_y$ can be estimated once seed values for $x_c$, and $y_c$ are known. Then the MSE method will iteratively adjust the parameters to determine a best fit. The fitted $\sigma_x$ and $\sigma_y$ are used to estimate FWHM values using Equation 6-3S.

Other Issues. It is often desirable to know the spatial resolution of one particular component of an imaging system. For example, if the overall LSF of an imaging system is the result of convolving three component LSFs we have

$$\mathrm{LSF}(x) = \mathrm{LSF}_1(x) \otimes \mathrm{LSF}_2 \otimes \mathrm{LSF}_3 \qquad \text{6-4S}$$

We also can represent the system response as modulation transfer functions (MTFs) in the frequency domain as

$$\mathrm{MTF}(u) = \mathrm{MTF}_1(u) \cdot \mathrm{MTF}_2(u) \cdot \mathrm{MTF}_3(u). \qquad \text{6-5S}$$

If $\mathrm{MTF}_1$ is what we desire and we know $\mathrm{MTF}_2$ and $\mathrm{MTF}_3$ (i.e., x-ray system screen MTF from data sheet and MTF of our digitizer) then we can estimate $\mathrm{MTF}_1$ as

$$\mathrm{MTF}_1(u) = \mathrm{MTF}(u) \cdot [\mathrm{MTF}_2(u)]^{-1} \cdot [\mathrm{MTF}_3(u)]^{-1} \qquad \text{6-6S}$$

This processing is called deconvolution and can also be done in the spatial domain. To avoid having to correct LSF and MTF for the effects of measuring components (screen, film, digitizer, slit width, etc.) it is recommended that we select these such that the width of each component is 1/10th or less of the FWHM you want to measure. This leads to a good approximation of FWHM which is easy to justify for the case when all component LSFs are Gaussian.

$$\begin{aligned} FWHM &= \sqrt{FWHM_1^2 + FWHM_2^2 + FWHM_3^2} \\ FWHM &= FWHM_1\sqrt{1 + \tfrac{1}{100} + \tfrac{1}{100}} \approx FWHM_1[1 + \tfrac{1}{2}(\tfrac{1}{100} + \tfrac{1}{100})] \\ FWHM &\approx 1.01\,FWHM_1 \end{aligned} \qquad \text{6-7S}$$

Model Problems. In clinical use the FWHM of a LSF may not be properly modeled by a single Gaussian. When the tails of the spread function are broader than expected (i.e. due to scatter) it may be more suitable to use simpler methods and assess both FWHM and FWTM as the indices of resolution.