CHAPTER 11: TEMPORAL FILTERING TECHNIQUES

11.1 Introduction

The primary problem in digital subtraction angiography is the presence of noise in the subtracted images. This situation arises because the opacification signal in these studies occupies only a small portion of the video signal which otherwise is dominated by the patient's anatomical structure.

For example, assume that the mask image I_m and opacification image I_o are acquired, each with a signal-to-noise ratio of 1000 to 1. We subtract the opacification image from the mask image to isolate the contrast-enhanced artery,

$$S = I_m - I_o \tag{11-1}$$

Therefore, the opacification in the artery occupies a small portion of the dynamic range of the video signal. Indeed, we can approximate the arterial opacification as providing a 1% contrast with respect to the maximum video signal V_{max}. Second, if the noise variances in the mask and opacification images are both equal to σ^2, then the noise variance in the subtracted image is

$$\sigma_s^2 = 2\sigma^2 \tag{11-2}$$

Therefore, if the signal-to-noise ratio of the unsubtracted images is

$$SNR = \frac{V_{\max}}{\sigma} = 1000 \tag{11-3}$$

then the signal-to-noise ratio of the subtracted images is

$$SNR_s = \frac{opacification\ \ signal}{\sigma_s} = \frac{0.01V_{\max}}{\sqrt{2}\sigma} = \frac{10\sigma}{\sqrt{2}\sigma} = 7.07 \tag{11-4}$$

Even though the mask and opacification images are acquired at a signal-to-noise ratio of 1000:1, most of the signal is contributed by the patient's anatomy. As a result, after subtraction, the arterial image has a low signal-to-noise ratio of approximately 10. In actual clinical practice, the signal-to-noise ratio of the subtracted angiogram is reduced even more by the existence of bright spots or limitations in the performance of the video camera. This calculation and these comments emphasize that digital subtraction angiography is an examination in which the diagnostic task will be limited by noise. The noise-limited nature of the images forces us to consider various methods to increase the signal-to-noise ratio of the resulting subtracted angiograms.

As we discussed in the previous chapter, there are several ways in which the signal-to-noise of the study can be improved. The first is utilization of an image intensifier with the highest possible detection efficiency, a video camera with the lowest possible electronic noise, and an analog-to-digital converter with an adequate number of levels so that it doesn't introduce quantization errors into the image data. The second is the use of bolusing so that bright spots do not compromise the opacification signal in adjacent dark regions. The third is proper adjustment of the camera aperture so that the images are acquired with a maximal video signal but at the smallest radiation exposure consistent with the quantum statistical requirements of the study. The last method is that of "image integration" in which multiple images containing the opacification signal are added (averaged) to reduce the effect of random noise (from both electronic and quantum statistical sources) while preserving the opacification signal.

The purpose of this section is first to present the basic theory of temporal filtering techniques used in digital subtraction angiography. We then will present three different temporal filtering techniques (mask mode subtraction, matched filtering, and recursive temporal filtering), describe them mathematically, and discuss the advantages and disadvantages of each.

11.2 Mathematical Conventions

In this section, we will represent the acquired images as I_i where "i" is the image index having only integral values, while S represents the subtracted image. This is a mathematical simplification since I_i represents an array of digital numbers. For example, the expression

$$S = I_2 - I_1 \qquad (11\text{-}5)$$

means that each pixel in S is calculated by subtracting corresponding pixels in I_1 from those in I_2. Similarly if h_i is a constant, the expression h_iI_i is the image produced when each pixel in the image I_i is multiplied by the constant h_i.

11.3 Temporal Filtering Theory

Assume we have a set of N images which we represent by I_i where i = 1, 2,..., N. The images have both a static component representing the stationary anatomical background as well as a dynamic component that, in the case of angiography, represents the time-varying arterial opacification contributed by the injected iodinated contrast media.

We will choose constants h_i to combine the images I_i to form the integrated image S where

$$S = \sum_{i=1}^{N} h_i I_i \qquad (11\text{-}6)$$

If I_i is constant (i.e. I_i= I), representing anatomical structure in the image, i.e. with no contrast media, then we can remove the anatomical structure by choosing the constants h_i such that

$$\sum_{i=1}^{N} h_i I_i = I \sum_{i=1}^{N} h_i = 0 \qquad (11\text{-}7)$$

so that constant anatomy is removed by satisfying the condition that

$$\sum_{i=1}^{N} h_i = 0 \qquad (11\text{-}8)$$

The size of the remaining time-varying opacification signal depends, of course, on the choice of the constants h_i as well as the time-course of the iodine bolus as it passes through the artery. Second, we can calculate the noise variance (σ_s^2) of the filtered (subtraction) image assuming that all of the individual acquired images have the same noise variance σ^2. That is, since σ^2 is constant, then

$$\sigma_s^2 = \sum_{i=1}^{N} h_i^2 \sigma_i^2 = \sigma^2 \sum_{i=1}^{N} h_i^2 \qquad (11\text{-}9)$$

Our goal is to choose values for h_i's that minimize noise variance σ_s^2 and maximize the time varying component of the signal S (i.e. that due to a contrast agent).

11.4 Mask Mode Subtraction

The simplest, and probably the most widely used image processing technique is called mask-mode subtraction in which the opacification (post injection of iodine contrast) image I_o is subtracted from the mask image I_m (obtained before injection of iodine contrast), producing the subtraction image.

$$S = I_m - I_o \tag{11-10}$$

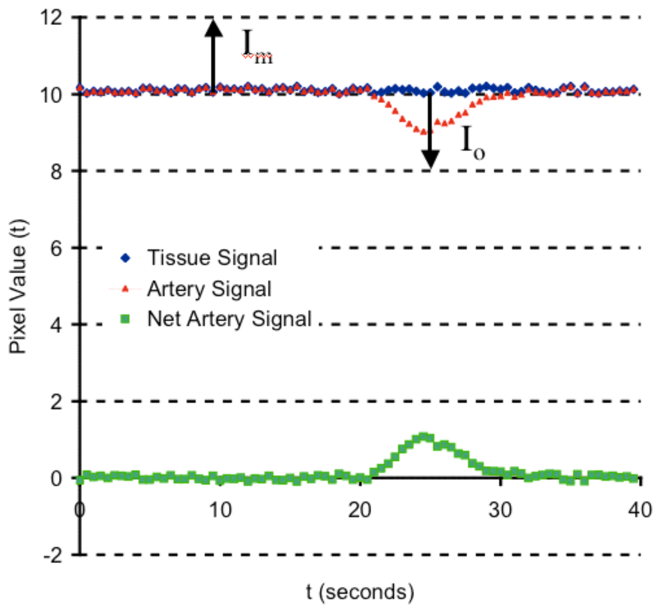


Figure 11-1. Simple mask mode subtraction is a process that subtracts an image obtained after opacification (I_o) from an image obtained before opacification (I_m). In integrated mask mode subtraction, multiple images are acquired before and after opacification and integrated (i.e, added) to decrease noise.

From Eq. (11-7) our filtering constants are $h_1 = +1$ and $h_2 = -1$ with the noise variance given by

$$\sigma_s^2 = 2\sigma^2. \tag{11-11}$$

When using a single acquisition the subtraction of each pixel value from the average pixel value acquired before the bolus injection (0-20 seconds red curve in Fig 11-1) produces a net time activity curve (bottom of Figure 11-1). When this processing is done for each pixel, the image series can be viewed as subtraction cine where the signal is approximately proportional to the concentration of contrast material. This removes stationary anatomy while retaining bolus signals. Viewing the subtraction cine allows physicians to evaluate the extent and time course of the contrast material as it passes through the arteries.

For integrated mask mode subtraction a mask image is formed as the sum of pre-injection images and an opacification image the sum of post-injection images. Simple subtraction (11-10) is then used.

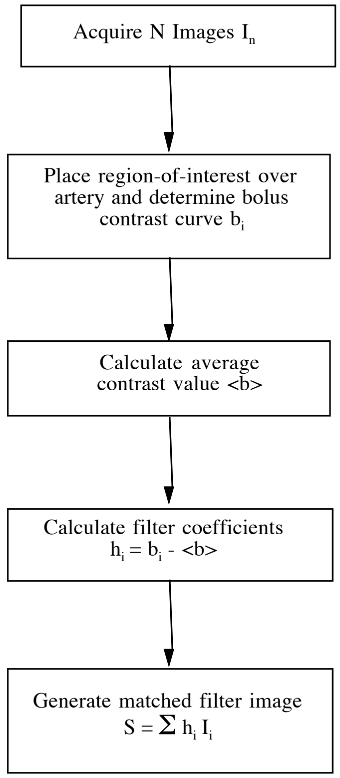
Note: All pixel values are the result of some form of scaling and logarithmic conversion. The sense of signals in Figure 11-1 is such that the contrast media decreases pixel values in the artery.

11.5 Matched Filtering

Both Kruger and Riederer utilized signal processing theory to propose a "matched filter" to maximize the signal-to-noise ratio of DSA images. In this technique, a signal is filtered temporally with a function having a shape matching the temporal response of the signal itself. Specifically, if the output waveform is expressed as the convolution of a filter with an input waveform containing a signal b(t) of known shape, the matched filter is proportional to b(-t). Alternatively, if the output is expressed as a correlation of a filter with the input waveform, the matched filter is proportional to b(t). It can be shown that the matched filter is the filter that maximizes the signal-to-noise ratio of the filtered signal.

We can extend this concept to maximize the signal-to-noise ratio of the opacification signal in DSA images. In particular, we assume that we are imaging an artery through which a contrast bolus represented by b(t) is passing, but which also contains the stationary patient anatomy. Regions containing stationary patient anatomy correspond to areas in the image having an ~ constant pixel value. Regions containing opacified arteries are represented by pixels containing both a constant anatomical component as well as the superimposed contrast bolus b(t) which is indexed in the imaging sequence as b_i.

MATCHED FILTERED ALGORITHM



Note: The matched filter algorithm is a post-processing technique. All images must be acquired before the filter is applied.

Figure 11-2. In matched filtering the weighting coefficients are equal to the difference between the bolus signal and its mean value.

We will begin by calculating the signal-to-noise ratio in matched filtering, and then will compare this to the signal-to-noise ratio of conventional DSA. We will form an image

$$S_{mf} = \sum_{i=1}^{N} h_i I_i \tag{11-12}$$

where we choose the filter constants h_i such that

$$h_i = b_i - \langle b \rangle \tag{11-13}$$

with

$$\langle b \rangle = \frac{1}{N} \sum_{j=1}^{N} b_j \tag{11-14}$$

In regions of the image where we are imaging stationary objects ($I_i = I$),

$$S_{mf} = \sum_{i=1}^{N} h_i I_i = I \sum_{i=1}^{N} h_i = I \left[\sum_{i=1}^{N} b_i - \sum_{i=1}^{N} \langle b \rangle \right] = I \left[N\langle b \rangle - N\langle b \rangle \right] = 0 \tag{11-15}$$

so that the matched filter does remove stationary anatomy from the processed images. In regions where we are imaging the contrast bolus,

$$S_{mf} = \sum_{i=1}^{N} h_i b_i = \sum_{i=1}^{N} \left[b_i - \langle b \rangle \right] b_i = \sum_{i=1}^{N} b_i^2 - \langle b \rangle \sum_{i=1}^{N} b_i$$
$$= N\langle b^2 \rangle - N\langle b \rangle^2 = N\left[\langle b^2 \rangle - \langle b \rangle^2 \right] = N b_{rms}^2 \tag{11-16}$$

Similarly if $S = \sum_{i=1}^{N} h_i b_i$ (11-17)

then the variance σ_s^2 in S is

$$\sigma_s^2 = \sum_{i=1}^{N} h_i^2 \sigma_i^2 = \sigma^2 \sum_{i=1}^{N} h_i^2 = \sigma^2 \sum_{i=1}^{N} \left(b_i - \langle b \rangle \right)^2 = N\sigma^2 b_{rms}^2 \tag{11-18}$$

Therefore the signal-to-noise ratio of the matched filtering technique is

$$SNR_{mf} = \frac{S_{mf}}{\sigma_s} = \frac{\sqrt{N} b_{rms}}{\sigma} \tag{11-19}$$

We will now compute the signal-to-noise ratio for conventional DSA. We will assume that we take the same N images, N/2 which are integrated to form a mask and N/2 which are integrated to form the opacification image (Figure 11.2, lower). The signal is

$$S_{DSA} = I_m - I_o = \sum_{i=1}^{\frac{N}{2}} b_i - \sum_{i=\frac{N}{2}+1}^{N} b_i = \frac{N}{2}\left[\langle b_m \rangle - \langle b_o \rangle \right] \tag{11-20}$$

while the noise is

$$\sigma_{DSA} = \sqrt{N}\sigma \tag{11-21}$$

so that the signal to noise ratio is

$$SNR_{DSA} = \frac{\sqrt{N}}{2\sigma}\left[\langle b_m \rangle - \langle b_o \rangle \right] \tag{11-22}$$

and the ratio of the signal-to-noise ratios obtained with matched filtering to that obtained with integrated mask mode subtraction is

$$\frac{SNR_{MF}}{SNR_{DSA}} = \frac{\dfrac{\sqrt{N}\ b_{RMS}}{\sigma}}{\dfrac{\sqrt{N}\left[\langle b_m \rangle - \langle b_o \rangle \right]}{2\sigma}} = \frac{2 b_{RMS}}{\langle b_m \rangle - \langle b_o \rangle} \tag{11-23}$$

So whenever twice the b_{RMS} exceeds the difference in mean values in the mask and opacified region then matched filtering provides an improvement in SNR. This was explored with the data from Figure 11-1 yielding an SNR improvement of approximately 2X. For an ideal bolus, where the signal is a square wave, integrated mask mode subtraction and matched filtering give the same result, otherwise matched filtering should improve the SNR.

An alternative approach to improve SNR in DSA is to fit the bolus curve to an analytical model over a range such as the data from 20-40 seconds in the net arterial curve of Figure 11-1. One approach uses a gamma variate function to model the different rising and falling characteristics of the bolus curve as follows:

$$b(t) = At^{B}e^{-Ct} \tag{11-24}$$

A, B, and C are parameters that are adjusted during the fit. Riederer has suggested the values B=4.0 and C = 0.9 sec^{-1}, with A chosen to give b(tmax) = 1 for a bolus curve with maximum value set to unity. The maximum of b(t) is achieved when its derivative is equal to 0 so that

$$\frac{db}{dt} = Ae^{-Ct}\left[-Ct^{B} + Bt^{B-1}\right] = 0 \tag{11-25}$$

showing that the maximum is obtained at

$$t_{\max} = \frac{B}{C} = \frac{4.0}{0.9\,\mathrm{sec}^{-1}} = 4.44\,\mathrm{sec} \tag{11-26}$$

Using the values B = 4.0, C = 0.9 sec^{-1} , and t_{max} = 4.44 sec, and satisfying the condition that

$$b(t\max) = b(4.44\,\mathrm{sec}) = 1 = A(4.44)^{4}e^{-0.9*4.44} = 7.146A \tag{11-27}$$

we find that

$$A = 0.140 \tag{11-28}$$

Thus, a typical contrast bolus curve for t in seconds is represented by the equation

$$b(t) = 0.140t^{4}e^{-0.9t} \tag{11-29}$$

This equation was in fact used to model the data seen in Figure 11-1. Using the range over just the bolus to fit an assumed model is attractive, but may not work as well in non-arterial regions. Fitting of theoretical models of the bolus activity leads to fitted values of A, B, and C. If each pixel is fitted with a model curve, then images created from the model curves can be used to view arterial dynamics.

<u>Selection of the ROI for determining b(t)</u>.

1. To focus on a particular artery, position the ROI within the artery. Placement will determine timing of b(t) and will produce best results near the ROI.

2. To improve the noise for the arterial ROI it can be enlarged slightly along the artery, but reduces the peak value of b(t) compared with smaller ROI.

3. Let each pixel be an independent ROI. No ROI tracing is needed. This will lead to a large number of b(t) curves (one per pixel) which can enhance all arteries. More noisy than methods 1 & 2 but still suppresses anatomy.

11.6 Recursive Filtering

Mask mode subtraction requires highly stable components including the video camera as well as the x-ray system. In 1981, Kruger and Gould independently proposed a technique that could accept the video signal from a standard fluoroscopy system with which the images were acquired continuously. This method, called time-domain filtering or recursive filtering, avoids the requirement for timing control of the x-ray system and can be simply added to a fluoroscopy system. In addition, this technique is relatively immune to small instabilities in the video camera or x-ray system which otherwise would seriously affect images produced by mask mode subtraction.

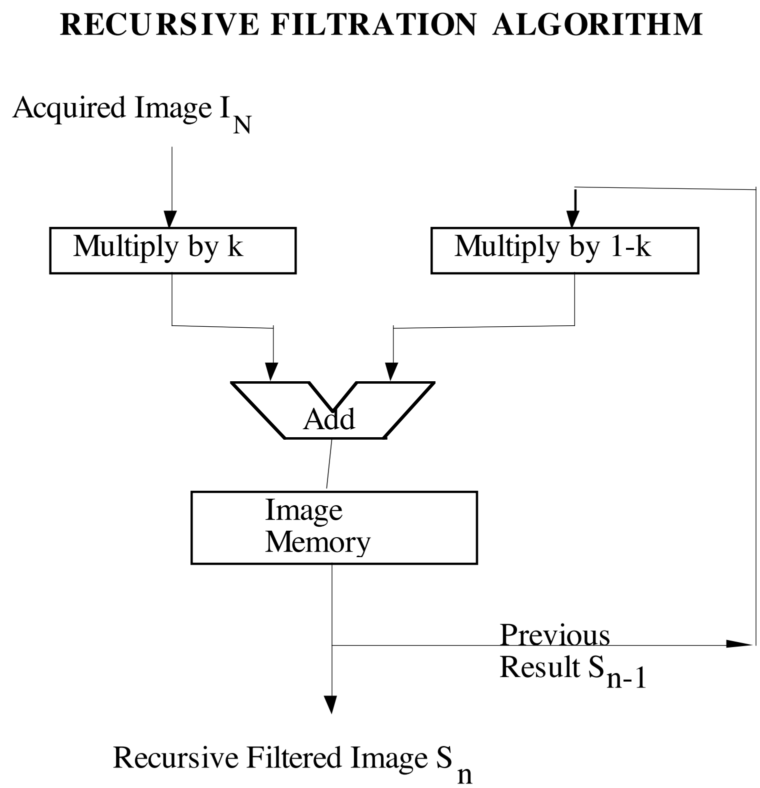


Figure 11-3. The recursive filter creates an output image that is a weighted sum of prior images in a time sequence.

The methods of Kruger and Gould both use the recursion relationship

$$S_n = kI_n + (1-k)\, S_{n-1} \quad n = 1, 2, 3, ... \tag{11-31}$$

where

k is a constant, $0 < k < 1$
I_n is the n^{th} video frame of an imaging sequence
S_n is the n^{th} image obtained from recursive filtering process

If the first video frame of the imaging sequence is

$$S_o = kI_o \tag{11-32}$$

then the following frames are generated according to equation 11-31. For example,

$$S_1 = kI_1 + (1\text{-}k)\, S_o = kI_1 + k\,(1\text{-}k)\, I_o \tag{11-33}$$

$$S_2 = kI_2 + (1\text{-}k)S_1 = kI_2 + k\,(1\text{-}k)I_1 + k(1\text{-}k)^2\, I_0 \tag{11-34}$$

$$S_3 = kI_3 + (1\text{-}k)S_2 = kI_3 + k(1\text{-}k)I_2 + k(1\text{-}k)^2\, I_1 + k(1\text{-}k)^3\, I_o \tag{11-35}$$

and in general

$$S_N = \sum_{m=0}^{N} k(1-k)^m I_{N-m} \tag{11-36}$$

We now will evaluate the response of recursive filtering to a constant signal (stationary anatomy), to transient signals (the iodine contrast bolus), and to random noise.

11.6.1 Response to Constant Signal

If all of the video frames are identical (with the exception of noise) as in the case of stationary patient anatomy, we will show that in the limit of a large number of video frames (i.e. $N \to +\infty$), then the recursive filter returns this constant input. Assume that we have identical frames where

$$I_i = I \quad \text{for} \quad i = 0, 1, 2, \ldots N \tag{11-37}$$

so that the recursive filter generates the new image following the N^{th} image as

$$S_N = kI_N + k(1\text{-}k)I_{N-1} + k(1\text{-}k)^2 I_{N-2} + \ldots + k(1\text{-}k)^N I_0 \tag{11-38}$$

which for this special case, equals

$$S_N = kI\left[1 + (1-k) + (1-k)^2 + \ldots + (1-k)^N\right] \tag{11-39}$$

The geometric sequence in Eq (11-39), with a ratio of (1-k), approaches $1/[1-(1-k)] = 1/k$ as $N \to +\infty$, therefore for large N

$$S_N = I \tag{11-40}$$

so that the recursive filter retains the constant input signal in its output. Therefore, if constant I suggest constant patient anatomy, the recursive filter retains that anatomy in the final filtered image, <u>unlike the two previous filtering methods</u>.

11.6.2 Response to Noise

We now will calculate the noise variance of the filtered image S_N. In this calculation, we will assume that image I_i has a similar level of noise variance that we will denote by σ^2. Since the recursive filter generates an image S_N where

$$S_N = k\, I_N + k(1\text{-}k)I_{N-1} + k(1\text{-}k)^2\, I_{N-2} + \ldots + k(1\text{-}k)^N\, I_o \tag{11-41}$$

and from propagation of errors we know that

$$\sigma_s^2 = \left(\frac{\partial S_N}{\partial I_N}\right)^2 \sigma_N^2 + \left(\frac{\partial S_N}{\partial I_{N-1}}\right)^2 \sigma_{N-1}^2 + \ldots + \left(\frac{\partial S_N}{\partial I_0}\right)^2 \sigma_0^2 \tag{11-41}$$

$$\sigma_s^2 = k^2\sigma_N^2 + k^2(i-k)^2\sigma_{N-1}^2 + k^2(i-k)^4\sigma_{N-2}^2 \ldots k^2(i-k)^{2N}\sigma_0^2$$

with all $\sigma_N \sim \sigma$ this becomes

$$\sigma_s^2 = k^2\sigma^2\left[1 + (1-k)^2 + (1-k)^4 \ldots + (1-k)^{2N}\right] \qquad (11\text{–}42)$$

and as N → + ∞ the geometrical sequence simplifies to

$$\sigma_s^2 = \frac{k^2\sigma^2}{1-(1-k)^2} = \frac{k\sigma^2}{2-k} \qquad (11\text{-}43)$$

which becomes infinitesimal for small values of the recursion constant k. Thus, the noise σ_S becomes negligible as k approaches zero.

For imaging coronary arteries, Kruger has suggested using a value of k = 1/16. This means that the noise level for recursive filtration is

$$\sigma_s^2 = \frac{\frac{1}{16}\sigma^2}{2-\frac{1}{16}} = \frac{1}{31}\sigma^2 \qquad (11\text{-}44)$$

so that the standard deviation of the noise in the recursively filtered image is reduced to approximately 18% that in the individual component images. This gain is made at the expense of higher patient exposure since in theory, it is obtained only after an infinite number of images have been filtered and combined by the recursive algorithm. In actual use, and as we will see in the following discussion, the recursive filter has a finite temporal response so that only a finite number of images need to be combined.

11.7 Response to Transient Signals

One of the more interesting properties of the recursive filter is its response to a transient signal such as the passage of a iodine contrast bolus through an artery. In this section we show that the output of a recursive filter to a transient (i.e time-varying) input signal can be approximated by convolving the input time series with an exponential function. If the output of the recursive filter is

$$S_N = kI_N + k(1-k)I_{N-1} + k(1-k)^2 I_{N-1} + \ldots + k(1-k)^N I_0 \qquad (11\text{-}45)$$

that can be expressed as weighted sums of individual image terms as

$$S_N = s_0 + s_1 + s_2 + \ldots s_N$$

where s_m is the mth term of the recursive filter output

$$s_m = k(1-k)^m I_{N-m} \qquad (11\text{-}46)$$

Equation 11.46 can be expressed as

$$s_m = k\left[(1-k)^{\frac{1}{k}}\right]^{mk} I_{N-m} \qquad (11\text{-}47)$$

and for small values of k,

$$(1-k)^{\frac{1}{k}} \approx e^{-1} \qquad (11\text{-}48)$$

For example with k=1/16 then $(1\text{-}1/16)^{1/16}$ = 0.3461 and e^{-1} = 0.3679. We can therefore approximate s_m using an exponential term as

$$s_m \approx ke^{-mk} I_{N-m} \tag{11-49}$$

Therefore, the filtered image (S_N) is a discrete convolution of temporal functions,

$$S_N = \sum_{m=0}^{N} ke^{-mk} I_{N-m} = h \otimes I \tag{11-50}$$

Thus, the recursive filter operation performs low-pass filtering of the input image with a filter of the form

$$h(m) \approx ke^{-mk} \tag{11-51}$$

The effect of filtering is to retain constant input signals (i.e. anatomical structures) since h(m) sums to ~ unity, and suppress noise through image integration.

The response of a recursive filter to the time varying bolus requires a closer look to its temporal response. An example can help with this. If m is the frame number in a set of images acquired with a fluoroscopic system, and if the images are acquired at 1/30 sec intervals (i.e. 30 per second), then

$$m = 30t_m \tag{11-52}$$

where t_m is the time in seconds of the m^{th} frame, then the filter function can be written as

$$h(t_m) \approx ke^{-30kt_m} \tag{11-53}$$

This shows that the approximated recursive filter function $h(t_m)$ has a time constant $\tau = 1/(30k)$ sec (Figure 11.3). As mentioned before, Kruger found that a recursion constant having a value of k = 1/16 was useful for coronary arteriography. For this example, this provides a filter function with a time constant of $\tau = 16/30 = 0.53$ sec. The temporal response of a recursive filter (Eq. 11-53), when reflected, illustrates the temporal change in filter weighting (Figure 11-4 bottom). The filter image S_N receives the greatest weight with weight dropping by 37% after one time-constant, 13% after two time constants, and 5% after 3 time constants. When the temporal history prior to image I_N is similar to that of reflected temporal filter response then a larger signal will be seen.

Motions that are slow relative to the filters time constant show little effect, while those that change too rapidly will be averaged with the earlier times. A time constant of $\tau = 0.34$ sec will help suppress motion artifacts due to breathing, will blur cardiac motion over a fraction of a cardiac cycle, and will average out the bolus passing through the artery for the same period of time. A shorter time constant would also be usable, but would integrate fewer frames and increase their noise contribution. If the time constant were longer, say $\tau = 1$ sec, the coronary artery would be blurred since the images would be integrated over an entire cardiac cycle.

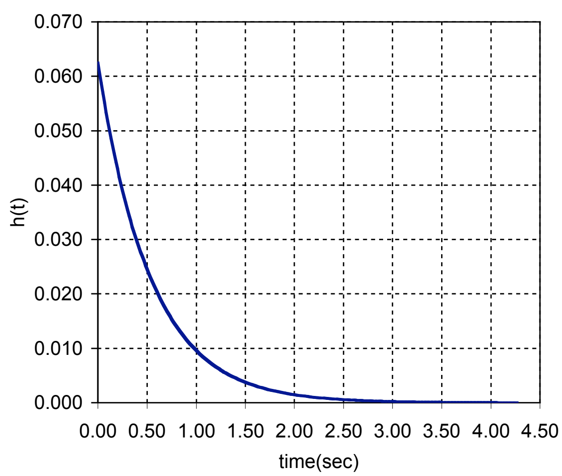


Figure 11.3. Recursive filter $h(t_m)$ in Eq 11-53 with $\tau = 0.53$ sec.

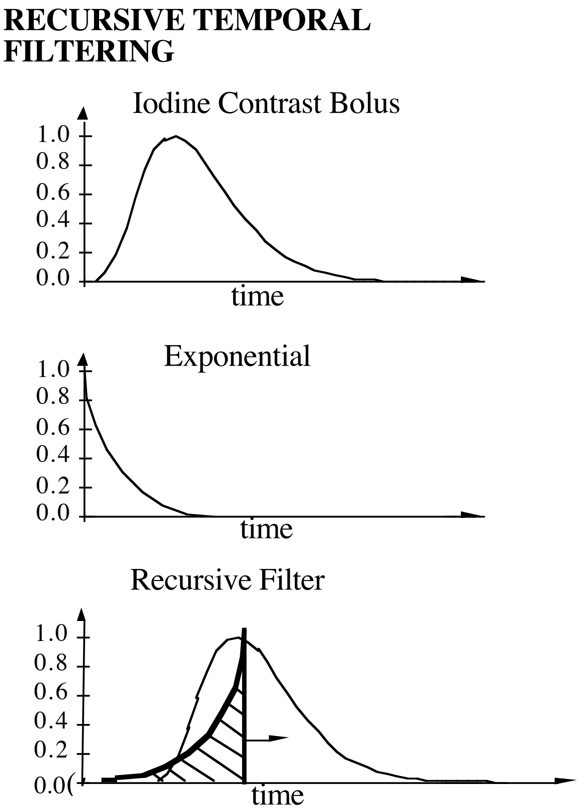


Figure 11-4. In recursive filtration, an image is multiplied by a coefficient k, then is added to the previous recursively filtered image weighted by the coefficient 1-k. These terms are added to form the next recursively filtered image

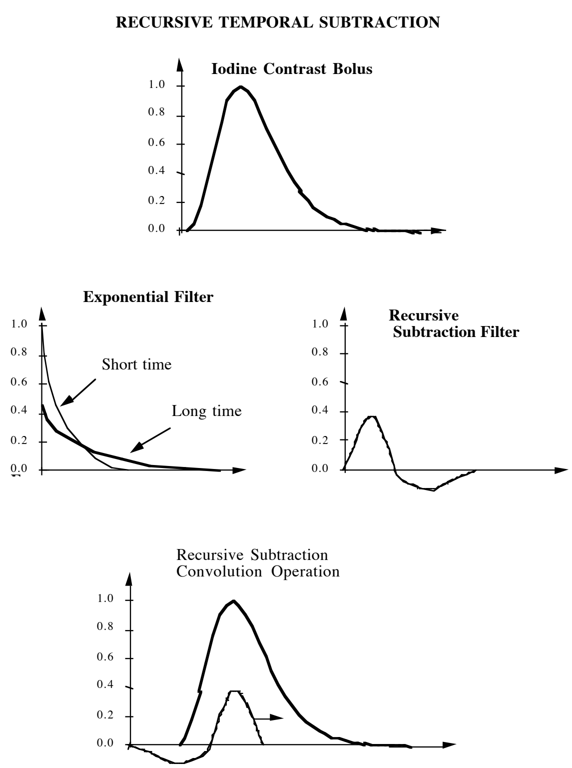


Figure 11-5. Recursive filtration is equivalent to convolving (in the temporal domain) the sequence of images by an exponential filter function

Band-Pass Temporal Subtraction Using Recursive Filters

As we have shown, the recursive filter retains stationary anatomical structure. In many cases, we want to suppress stationary anatomy as we do in mask mode subtraction. Generating two recursively-filtered images with different time constants from the same data set can accomplish this. The recursive image with a long time constant could be subtracted from the recursive image with a short time constant. Objects having motions faster than that corresponding to the short time constant would be suppressed by the recursive filter, while those stationary objects retained in both recursive images would be removed by subtraction. This is the concept behind the recursive band-pass filter that simultaneously eliminates stationary anatomy, reduces noise, and suppresses the appearance of structures, such as the heart, which are moving rapidly in the image.

We begin this discussion by looking at Figure 11-6 showing the various temporal frequency components in a fluoroscopic imaging sequence following the intravenous injection of a contrast agent into the circulatory system. The delta function at zero temporal frequency represents the patient's anatomy that we presume is stationary (obviously a gross simplification unless the person is dead or sleeping (during a medical imaging class). The contrast bolus passing through the artery is imaged over the period of 3 to 10 sec and therefore is represented by the temporal frequency components in 0.1 to 0.3 Hz range. Finally, the subject's heart beats every 1 to 2 seconds representing the temporal frequency in the highest range of this example.

To image the contrast bolus as it passes through the artery, but to eliminate stationary patient anatomy and suppress cardiac motion, we can design a temporal band-pass filter. This is formed using the recursive filter to obtain a low pass filter that removes spatial frequency components above 1 Hz (where cardiac motion resides). From this, we can subtract another recursive-filtered image that retains only patient anatomy. This has the effect of a composite band-pass filter that removes both the patient anatomy as well as any structures subject to cardiac motion and retains information about the contrast bolus passing through the artery.

Temporal Frequency Components in a Coronary Angiogram

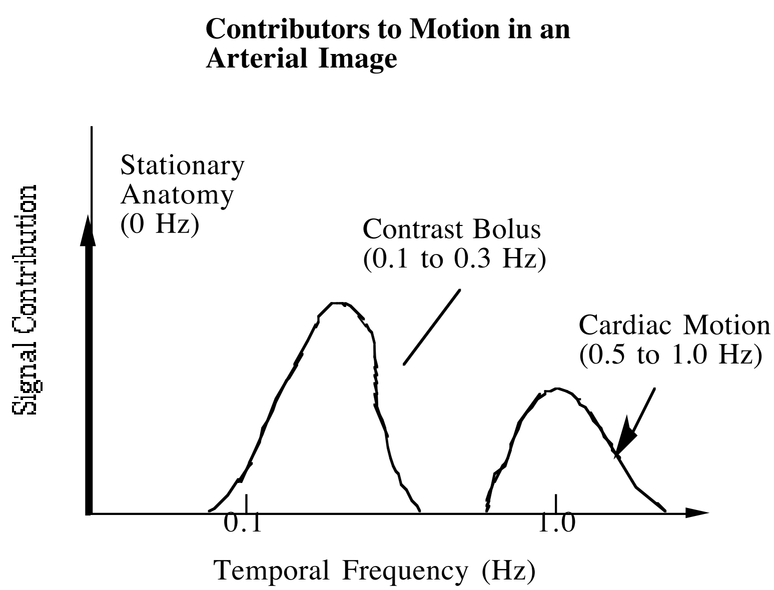


Figure 11-6; In recursive temporal subtraction, the filter function is formed by subtracting a filter with a long time constant with one with a short time constant. The recursive subtraction filter then is convolved with the set of images in the temporal domain. This can isolate the contrast bolus as it passes through the circulatory system with a given range of temporal frequencies

Given the frequency distributions in Figure 11-6 for stationary anatomy, contrast bolus, and cardiac motion, it is instructive to look at the frequency response of recursive temporal filters. Using h(t) derived from Eq. 11-53 as the temporal domain filter response we can calculate the normalized |H(f)| for a the recursive filter as

$$|H(f)| = \left(\frac{1}{1+(2\pi f\tau)^2} \right)^{1/2} \qquad \text{Eq 11-54}$$

The bandwidth (BW) of this recursive filter $\frac{\sqrt{3}}{2\pi\tau} \approx \frac{0.276}{\tau}$ so when τ = 0.53 sec the BW = 0.52 cycles/sec. When the spatial version of individual recursive filters are subtracted as seen in Figure 11-5 the net frequency response is the difference in frequency response of the filters. Figure 11-7 shows how subtraction of two recursive filters can result in a bandpass type filter. The |H1|-|H2| filter has zero magnitude at f=0 so it suppresses stationary anatomy (f=0). This filter has its highest magnitude for a bolus (f=0.1-0.3 cycles/sec) and attenuates higher frequencies such as those associated with cardiac motion.

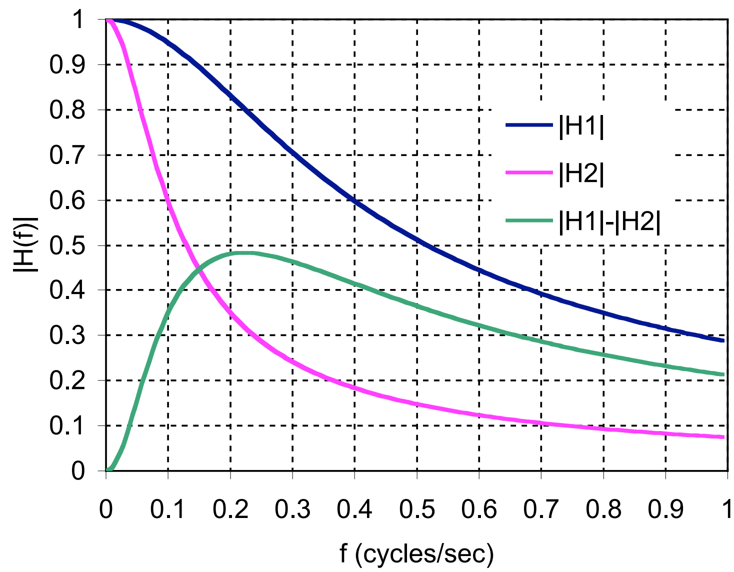


Figure 11-7. Magnitude frequency responses of recursive filters H1 and H2 with τ1 = 0.53 sec and τ2 = 2.13 sec. The |H1|-|H2| filter bandpass response peaks in the 0.1-0.3 cycles/sec range

While it is instructive to review recursive filters, medical image processing software for temporal filtering can developed using high-level scientific programming applications such as Matlab. This topic is explored further in our course on Medical Image Processing.

CHAPTER 11: HOMEWORK PROBLEMS

1. A radiologist brings you a set of 600 digitally subtracted video frames, stored on video tape, and obtained during an angiographic study following a single injection of contrast agent. The radiologist comments that the artery is barely visible in images and asks you if you have any "image processing tricks" to recover the images.

   After viewing the tape you realize first that the digital subtraction process has already removed the stationary anatomical information. You also discover that the images are very noisy and you decide to try a temporal matched filter to integrate the images to improve the signal-to-noise ratio. You place a large region-of-interest over the image and quantitate the total opacification in each frame I_i to obtain the contrast bolus curve with the individual values denoted by b_i .

   a. The matched filtering technique described in this chapter is used for processing <u>unsubtracted</u> images and is designed to suppress stationary anatomical information. For the image sequence given to you by the radiologist, the stationary anatomical information has already been removed by digital subtraction. Specify a matched filter for this new image sequence (i.e. specify the coefficients h_i), which maximizes the signal-to-noise ratio of the integrated image obtained from the digitally subtracted angiograms.

   b. If $<b^2>$ is the average value of the squared opacification values,

$$<b^2> = \frac{1}{N} \sum_{i=1}^{N} b_i^2$$

   show that the signal-to-noise ratio of the integrated image obtained with the matched filter for N images frames is

$$SNR_{mf} = \frac{\sqrt{N <b^2>}}{\sigma}$$

   c. We can compare the matched filter technique with an unweighted image integration technique where the image frames are simply added together without any constant multiplication. If <b> is the mean value of the opacification values

$$<b> = \frac{1}{N} \sum_{i=1}^{N} b_i$$

   show that this technique produces an integrated image having a signal-to-noise ratio of

$$SNR_{uw} = \frac{\sqrt{N} <b>}{\sigma}$$

   We would like to estimate the improvement in the signal-to-noise ratio produced by the matched filter in comparison to a single image and to unweighted temporal image integration. To do this, we will use the contrast bolus curve function given in the lecture notes

$$b(t) = 0.140\ t^4\ \exp(-0.9\ \text{sec}^{-1}\ t)$$

   which we will sample 30 times per second for 20 seconds, for a total of 600 frames. Furthermore, while the following calculations can be easily performed with a spreadsheet, you may also estimate the discrete samples with the continuous function b(t) given above, and estimate the sums with integrals using the formula

$$\int x^m e^{ax}\, dx = e^{ax} \sum_{n=0}^{m} (-1)^n \frac{m!\, x^{m-n}}{(m-n)!\, a^{n+1}}$$

d. Prove that the maximum value of b(t) is b_{max} = 1 at t = 4.44 sec. Use this result to calculate the improvement in the signal-to-noise ratio obtained with the matched filter in comparison to that obtained from the best single frame in the entire image sequence.

e. Calculate the percentage difference in signal-to-noise ratios obtained with the matched filter in comparison to unweighted temporal integration of the digitally subtracted angiograms.

2. We want to design a image processor capable of video-rate subtractive imaging with two recursive filters. Let I_i represent the i^{th} (logarithmically transformed) unsubtracted image frame received by the image processor. Furthermore, the n^{th} subtraction image S_n is defined by

$$S_n = U_n - V_n$$

U_n and V_n are the recursively -filtered images defined by

$$U_n = k_1 I_n + (1 - k_1) U_{n-1}$$

and

$$V_n = k_2 I_n + (1 - k_2) V_{n-1}$$

where $k_1$and k_2 are the recursion constants which define the temporal response of the filter.

a. Show that the n^{th} recursively filtered subtraction image can be written explicitly as

$$S_n = \sum_{m=0}^{n} \left[k_1 (1 - k_1)^m - k_2 (1 - k_2)^m \right] I_{n-m}$$

b. For large values of n (i.e. in the limit as $n \to +\infty$), assuming that the unprocessed images I_i each have a noise variance equal to σ, show that the noise variance σ_s^2 in the recursively subtracted image is given by

$$\sigma_S^2 = \sigma^2 \left[\frac{k_1}{2 - k_1} - \frac{2 k_1 k_2}{k_1 + k_2 - k_1 k_2} + \frac{k_2}{2 - k_2} \right]$$

c. Draw a block diagram for an image processor that would allow you to perform the subtractive recursive filter described in this question.

d. The unprocessed image sequence I(t) which is acquired at a rate of 30 frames a second. Using the partial results given in the lecture notes, show that the recursively filtered subtraction image S(t) can be represented as the temporal convolution

$$S(t) = I(t) \otimes \left[\frac{1}{30\tau_1} \exp\left(-\frac{t}{30\tau_1} \right) - \frac{1}{30\tau_2} \exp\left(-\frac{t}{30\tau_2} \right) \right]$$

where τ_1 and τ_2 are time constants obtained from the recursion constants

$$\tau_1 = \frac{1}{30 k_1} \text{ sec and } \tau_2 = \frac{1}{30 k_2} \text{ sec.}$$

e. For $k_1 = 1/16\ \text{sec}^{-1}$ and $k_2 = 1/32\ \text{sec}^{-1}$, graph the temporal filter function (as a function of time) derived in part d of this question. Intuitively discuss the temporal response of this filter on the image sequence with respect to stationary anatomy, the contrast bolus from an intravenous injection, and cardiac motion. Calculate the noise variance of this image using the equation you derived in part b of this question.