CHAPTER 10. DIGITAL SUBTRACTION ANGIOGRAPHY (DSA)

10.1 Introduction

In this chapter we will discuss a technique that is known as digital subtraction angiography but which is also referred to as digital radiography, digital fluoroscopy, and photoelectronic imaging. We will use these terms interchangeably although the term digital subtraction angiography refers specifically to techniques which subtract two images that are obtained before and after contrast media is administered to the patient for the purposes of studying blood vessels (angiography). The more general term, digital radiography encompasses the use of all digital electronic techniques in x-ray imaging. According to some writers this term also includes the use of x-ray computed tomography (CT), although digital radiography in this chapter will refer only to those techniques in which digital electronics are used to acquire planar rather than tomographic images. In addition, we will concentrate on systems that use image intensifiers viewed by video cameras since these systems provide helpful illustrative examples.

10.2 System Design for a Digital Radiograph System

A general system diagram for a digital radiographic system is given in Figure 10-1. At the heart of this system is a digital image processing system which acquires images from a video camera and provides timing signals to both the x-ray generator and the image acquisition system to control the flow of data from the x-ray source into the image processor.

The image acquisition process begins when timing signals, delivered to the x-ray generator under computer control, initiates the production of x-rays which are transmitted through the patient and received by the image intensifier. An aperture, placed between the image intensifier and the video camera, controls the amount of light delivered to the camera. This manages the signal-to-noise ratio of the acquired image as will be discussed later in this chapter. A video camera receives the light image from the image intensifier and converts it to an electronic video signal which is delivered to the image processor in analog form. The image processor digitizes the image, stores it in memory, and makes it available in digital form for subtraction with another image set acquired at a different time or at a different energy. The basic components of the imaging system including the x-ray tube and x-ray generator, the image intensifier, and the video camera are similar to but must be of higher quality than those used in conventional fluoroscopy to ensure proper synchronization and match between analog and digital components.

A common algorithm using digital radiographic systems is temporal subtraction (Figure 10-2). In this technique, dynamic images of the patient are acquired at a rate of 1 exposure per second or more. A contrast agent is injected into the patient either intravenously or intra-arterially. A second set of dynamic images is acquired after the contrast agent flows into the area being imaged. The unopacified images (no contrast) are subtracted from the opacified images with the subtraction process isolating the signal (which is present only in the opacification image), removing the static anatomical structures that
are common to both the opacified and unopacified images. The elimination of background structures makes the arteries visible in the subtraction image even when they are not visible or barely visible before subtraction.

10.3 Imaging System Components

10.3.1 Image Intensifier

The subtraction algorithm assumes that the patient's anatomy is similar or identical in both the mask and contrast images. The video camera, the x-ray tube, and the other system components must be stable enough to ensure this equality so the anatomical structure can be subtracted. To preserve the contrast available in the radiographic image, the image intensifier must have a high contrast ratio and the analog-to-digital conversion should provide sufficient spatial sampling to preserve the resolution of the image intensifier.

10.3.2 Light Aperture

A light aperture (Figure 10-3), similar to those found on single-lens reflex cameras, is placed behind the output phosphor of the image intensifier to control the amount of light reaching the video camera for a given exposure rate. A small aperture requires a greater radiation exposure to deliver an adequate light level to the video camera, decreasing the effect of quantum noise and producing a better overall signal-to-noise ratio in the image. Conversely, a large camera aperture is used when one wishes to minimize patient exposure in cases where quantum noise does not limit the diagnostic information in the image.

10.3.3 Video Camera

One of the most critical components in the imaging chain for our example digital radiographic system is the video camera. The basic function of the video camera is to produce an analog electronic signal that is proportional to the amount of light received by the target of the camera.

Figure 10.4, Schematic Of Video Camera

A schematic diagram of a video camera is given in Figure 10-4. The photoactive element in the camera is the video target that changes in electrical conductivity when exposed to light. Scanning an electron beam across sequential lines of the video target, with the deposited electronic charge passing through
the target to form an electrical current, creates the video signal. Regions where the target has been exposed to high light levels produce a high conductivity, and thus a large current. Regions of the target that have been exposed to low levels of light produce a lower conductivity and thus a smaller video camera current. The resulting signal is a measure of the light level input to the video target. The information is read out serially as the electron beam is swept over the target to generate an analog video signal. The video signal is time-varying signal that encodes the two-dimensional light image at the target as a temporal record. Time points in the video signal correspond to spatial locations within the light (and x-ray) image.

The video target can be read out in one of two different ways. In video cameras used in the broadcast industry, the electron beam is scanned across the target in 262 1/2 passes across the area of the target. The resulting 262 1/2 lines form an image of the target, which is called a video field (Figure 10-5). The video field is produced once every 1/60 of a second. During the next field, the electron beam scans along lines located between two lines of the previous field. Therefore, the second field is acquired with lines interlaced between those of the first field. The two fields are called even and odd fields with each field comprising 1/60 of a second, and two fields make one frame acquired in 1/30 of a second that is comprised of 525 lines. This scan choice was chosen by the broadcast industry to reduce bandwidth during transmission while avoiding flicker in the viewed video image. However, interlaced scanning is not ideal for digital radiography. The basic problem of the interlaced mode is that the video fields are read out continuously. However, most video targets have a certain amount of lag so that even when they are exposed to a constant light level, it takes several video fields before the output signal is stable. Thus, in digital radiographic systems, after the x-ray beam is initiated, lag in the video signal produces images obtained in this early phase to be unstable. These early fields must be discarded, although this clearly is undesirable since it underutilizes the x-ray exposure delivered to the patient (Figure 10.6). This is partly due to the fact that parts of a field will receive more exposure than others following the onset of an x-ray exposure.

Progressive mode scanning resolves the problems with wasted exposure (Figure 10.7). When this mode is utilized, an image is stored in the target during a short x-ray exposure and completely read out while the x-ray beam is off. This approach eliminates the wasted x-ray exposure that is needed to bring frame to a readable level using continuous x-ray exposure and interlaced scanning. It does this by not mixing the processes of storage and readout from the video camera target.

There are other aspects of the video camera that are important in a digital
radiographic system. First, the size of the video signal should be directly proportional to the x-ray fluence delivered to the input phosphor of the image intensifier. Second, the video camera must have low lag. This means that an image acquired at one point in time by the video camera does not persist on its target for more than one readout period. This is especially important where rapidly moving objects, such as the heart, are being imaged by the digital radiographic system. A video camera that has the characteristics of good contrast linearity as well as low lag is a plumbicon target camera since it features a camera tube with a lead oxide \( \text{(PbO}_2 \) vidicon target. Another benefit of plumbicon cameras is that they have excellent noise characteristics in comparison to other types of video cameras, offering electronic signal-to-noise ratios in the range of 1000:1 and as high as 2000:1.

### 10.4 Digital Image Processor

The block diagram for a typical digital image processor is shown in Figure 10-8 & 9. The digital processor has basic functions illustrated in this diagram including (1) acquiring and digitizing the video images, (2) storing the digital images in memory, (3) performing arithmetic operations (subtraction, addition and constant multiplication) on the image data, (4) displaying the digital images on video monitors, and (5) storing the image data on magnetic media or an optical disk. The image processor also contains a microprocessor or system controller that controls the basic operations of the image processor, the x-ray generator, and other components, coordinating and controlling the operation of the digital imaging system.

**Figure 10.8.** Digital Image Processing System

**Figure 10.9.** DSA System Components
**Image Intensifier:**

- **CsI input phosphor** - Excellent x-ray absorption
- **Titanium Window** - High contrast ratio (low light scattering)
- **Spatial Resolution** - Can be relaxed for digital imaging

**Variable Light Aperture:**

- **Small aperture** - Greater patient exposure for improving SNR
- **Large aperture** - Lower patient exposure at lower SNR

**Video Camera:**

- **Camera type**: Vidicon with lead oxide (PbO₂) target ("Plumbicon")
- **Camera response**: Output signal proportional to light input signal
- **Temporal characteristics**: low lag
- **Signal-to-Noise Ratio**: 1000:1 to 2000:1
- **Scanning Modes**: Progressive scanning for best dose utilization
  Interlaced scanning also used in some cases

### 10.4.1 Analog-to-Digital Conversion

We will assume that an analog video image has been acquired by the x-ray system, image intensifier, and video camera (Figure 10.8). The analog signal is delivered to the image processor that provides some preprocessing to adjust the amplitude and level of the video signal to satisfy the input specifications of the analog-to-digital converter. The step size of the analog-to-digital converter must be selected so that it does not introduce additional noise to the image signal after digitization and generally is chosen to approximately equal the standard deviation of the electronic noise. Because the signal-to-noise ratio of most video cameras is approximately 1000:1, the entire range of the video signal must be covered by approximately 1000 digitization of quantization steps, corresponding to an analog-to-digital converter with 10 bits (1024 steps) to cover this analog signal range. The ADC controls conversion of the temporal analog video signals to sequence of digital numbers. The controlling microprocessor uses software to format this sequence of digital numbers such that they can be used to index image locations using a row, column scheme. The spacing between columns is determined by the sampling rate of the ADC which together with video scan rate determine the number of columns for the digitized image. The number of rows is set by the number of scan lines provided per video frame.

Many images are acquired in a 512 x 512-pixel matrix although some systems use a 1024 x 1024 matrix. The image matrix controls the sampling rate of the analog to digital converter. For example, if a 512 x 512 image matrix is used to digitize an image that is acquired over 1/30 of a second; the sample period of the analog-to-digital converter equals 1/30 of a second divided by 512². This period is approximately 100 nanoseconds, corresponding to a sampling frequency of about 10 megahertz. This sampling frequency limits the bandwidth of the system to approximately 5 megahertz and the analog preprocessing must include a low-pass filter to avoid aliasing thereby limiting the spatial frequency of the incoming video signal to 5 megahertz or less in this example.

### 10.4.2 Logarithmic Transformation

Following digitization, the image data are logarithmically transformed, meaning that the pixel values are replaced by their logarithm. The logarithmic transformation is required to remove stationary anatomical
structure during image subtraction. The logarithmic transformation can be performed on the analog signal prior to digitization with a logarithmic amplifier (i.e. a specialized operational amplifier). However, most imaging systems currently perform the logarithmic transformation following the analog-to-digital converter with a digital look-up table that simply replaces a digital value with a new value proportional to its logarithm.

10.4.3 Image Memory and Integration Feedback Loop

After logarithmic transformation and digitization of the incoming video signal, the image is stored in one memory of the image processor. Each pixel in the digital image is represented by a digital number having a minimum of 10 bits corresponding to the digitization range of the image-processor’s analog-to-digital converter.

Often more than one image is added ("integrated") to reduce noise and improve the SNR of the image. This averaging is provided by a feedback loop in which the incoming image is added to the contents of the previously stored image on a pixel-by-pixel basis. If the image processor has a 10 bit analog-digital converter, the image memories must have at least 4 more bits available corresponding to the range of 16 – 32 bits per pixel. This allows multiple images of the same area to be averaged or added to reduce the noise in the acquired images. Virtually all image processors have more than one memory plane and many have 3 image memories. This requirement is obvious in the case of digital subtraction angiography where the mask image is acquired in one memory and then subtracted from opacification image acquired in a second memory. Where a sequence of images is acquired, they all can be stored in digital memory in the image processor. However, this is very expensive and a more likely technique is to subtract the images, enhance the contrast signal and store the resulting subtraction image either on a high speed magnetic or optical disk.

10.4.4 Image Subtraction

In digital subtraction angiography, two images are acquired. The first is the "mask" image, which is obtained before contrast media is injected into the patient. The second is the "opacification" image, which follows injection of the contrast media and is obtained when the contrast bolus reaches the artery to be imaged.

The mask and opacification images can be modeled mathematically by assuming that the patient has a thickness $x_t$ and a linear attenuation coefficient of $\mu_t$. Before contrast media is injected into the patient, the photon fluence delivered to the image intensifier is

$$I_m = I_0 e^{-\mu_t x_t}$$  \hspace{1cm} (10-1)

The contrast media then is injected to opacify the artery. If the artery has thickness $x_i$ (where $x_i \ll x_t$) and has a linear attenuation of $\mu_i$, the image intensifier receives a photon fluence of

$$I_f = I_0 e^{-(\mu_i x_i + \mu_t x_t)}$$  \hspace{1cm} (10-2)
If \( \alpha \) is the conversion factor which relates the amplitude of the video signal to the photon fluence received by the image intensifier, the mask and opacification image signals produced by the video camera are

\[
I_m = \alpha I_0 e^{-\mu_I x_I} \quad (10-3)
\]

\[
I_I = \alpha I_0 e^{-(\mu_I x_I + \mu_I x_T)} \quad (10-4)
\]

We now will use equations 10-3 and 10-4 to demonstrate the difference between subtraction of the images without logarithmic transformation (linear subtraction) and subtraction of the images following logarithmic transformation (logarithmic subtraction).

### 10.4.5 Linear subtraction

Some of the early investigators of digital subtraction techniques used a linear subtraction algorithm to isolate the opacification signal. In linear subtraction, the opacification image is subtracted from the mask image without logarithmic transformation. If \( S_{\text{lin}} \) is the subtraction image, then linear subtraction produces an image having the form

\[
S_{\text{lin}} = I_m - I_I = \alpha I_0 e^{-(\mu_I x_I)} - \alpha I_0 e^{-(\mu_I x_I + \mu_I x_T)} = \alpha I_0 e^{-(\mu_I x_I)} \left[ 1 - e^{-(\mu_I x_T)} \right] \quad (10-5)
\]

If we assume a small iodination signal such that \( \mu I x_I << 1 \), then

\[
S_{\text{lin}} = \alpha (\mu_I x_I) I_0 e^{-(\mu_I x_I)} \quad (10-6)
\]

This shows that, using linear subtraction, the thickness of the iodine (\( x_I \)) is modulated by the patient thickness \( x_T \). Linear subtraction produces images that retain the unwanted patient anatomy still superimposed on the desired opacified arterial image.

### 10.4.6 Logarithmic subtraction

In comparison to linear subtraction, logarithmic subtraction does not retain stationary anatomical structure that may obscure the small signal contributed by the opacified artery. The mask and opacification image data are subtracted after they are digitized and logarithmically transformed. Mathematically, the logarithmic subtraction image \( S_{\text{log}} \) is

\[
S_{\text{log}} = \ln(I_m) - \ln(I_I) = [-\mu_I x_I] - [-\mu_I x_I - \mu_I x_T] = \mu_I x_T \quad (10-7)
\]

so that the resulting logarithmic subtraction signal is related to the iodine signal and is not affected by the patient thickness or the anatomy on which the opacified artery is superimposed. However, the noise level in subtraction images is always higher.

---

1 In x-ray CT, image density is measured in terms of "Hounsfield units", abbreviated HU, and named after one of the fathers of CT, Godfrey Hounsfield. Similarly, your author proposes that DSA images be measured in terms of "Mistretta units",
10.4.7 Image Display and Archival Memory

After processing, the digital images are delivered to a digital-to-analog converter that generates an analog video signal fed to a video monitor. The image data then can be displayed for examination by the radiologist.

The analog video signal also can be stored using a video cassette recorder or analog video disk recorder for archival purposes. The dynamic range of older technology (i.e. analog tape or disk recorders) was approximately 200:1, in comparison to the 1000:1 dynamic range of the video camera, which we previously discussed. However, newer digital video recordings preserve the dynamic range of the digital radiographic images.

For absolute fidelity of the image data however, digital image storage is the preferred method. These devices (including magnetic disk, magnetic tape, and optical disk), record the image data in digital form which virtually avoids the possibility noise being added by the recording media. Digital image storage is necessary when images are recorded before subtraction, and is useful when extensive processing (image integration or iodine quantification) will be performed. The principal disadvantages of digital storage are that it is considerably more expensive than analog storage, and that one requires a digital image processor for displaying the image.

10.5 Noise in Digital Subtraction Angiography

In Chapter 8 we discussed how quantum statistical noise $\sigma_q$, electronic noise $\sigma_e$, and digital quantization noise $\sigma_d$ all contribute to system noise, and if we design our digital subtraction angiography system correctly, quantization noise is negligible.

By way of summary, if a radiographic signal is composed of $N$ photons, then the uncertainty (i.e. standard deviation) in that signal is $\sqrt{N}$, since photon generation and attenuation behave according to Poisson statistics. Assuming that the camera produces a maximum video output of $V_{\text{max}}$ at $N_{\text{max}}$ photons, the video signal $V_N$ corresponding to $N$ photons is given by the proportionality relationship

$$\frac{V_N}{V_{\text{max}}} = \frac{N}{N_{\text{max}}} \text{ or } V_N = \frac{V_{\text{max}}}{N_{\text{max}}} N \quad (10-8)$$

Where both $N_{\text{max}}$ and $V_{\text{max}}$ are constants for any one system configuration. Therefore, the uncertainty in the video signal due to quantum statistical sources is

$$\sigma_q = \left( \frac{V_{\text{max}}}{N_{\text{max}}} \right) \sigma_N = \left( \frac{V_{\text{max}}}{N_{\text{max}}} \right) \sqrt{N} \quad (10-9)$$

abbreviated MU, named after one of the fathers of DSA, Chuck Mistretta. Further, we propose that the Mistretta unit, MU, be pronounced "moo" rather than "mew". This will avoid confusion with the Greek letter $\mu$ having the same spelling. This pronunciation also commemorates the fact that much of the early work in DSA was conducted at the University of Wisconsin, in the midst of America's Great Dairyland.
The electronic noise contributed by the video camera typically is characterized in terms of the camera's dynamic range, defined to be the ratio of the peak video signal \( V_{\text{max}} \) divided by the standard deviation \( \sigma_e \) of the video signal. If \( D \) is the dynamic range of a video camera, then the standard deviation \( \sigma_e \) of the electronic noise from the camera is given by

\[
\sigma_e = \frac{V_{\text{max}}}{D}
\]  

Finally, the quantization error or quantization noise is the error introduced into that analog signal when it is digitized. If \( \Delta \) is the width of the quantization step (i.e. the interval associated with the least significant bit of the analog-to-digital converter) where all analog values from \( \mu - \Delta/2 \) to \( \mu + \Delta/2 \) are equally likely and are converted into the value \( \mu \) by the analog-to-digital converter, then the variance of the quantization error is

\[
\sigma_\Delta^2 = \frac{\Delta^2}{12}
\]  

10.5.1 System Noise in Digital Subtraction Angiography

The system noise variance in digital subtraction angiography is obtained by adding the noise variance from each system component, assuming that these noise contributions are independent. In this calculation, we assume that the image system consists of a video camera viewing the output phosphor of the image intensifier, and that the quantization error contributed by the analog-to-digital converter is negligible.

The camera output contains a component \( V_q \) proportional to the exposure to the input phosphor as well as a time-varying term \( V_c \) arising from the "dark current" of the system.

\[
V = V_q + V_c
\]  

The uncertainty in the video output can be calculated using propagation of errors

\[
\sigma_v^2 = \sigma_q^2 + \sigma_e^2
\]  

showing that the contributions from the video dark current \( \sigma_e \) and quantum statistical sources \( \sigma_q \) add in quadrature to give the total noise \( \sigma_v \) in the video system.

Substituting the expressions derived above, we obtain the noise in the video signal in terms of the number of photons (\( N \)), i.e. number of photons per pixel, the dynamic range (\( D \)) of the video camera, the maximum video level (\( V_{\text{max}} \)), and the maximum number of photons:

Figure 10-11. In an ideal video camera, the video signal output is proportional to the level of light exposure up to a maximum saturation level (\( V_{\text{max}} \)). The video dynamic range is defined as the maximum video signal divided by the root mean square of the noise in the video signal.
of photons \( (N_{\text{max}}) \) corresponding to the maximum video level (Figure 10.11)

\[
\sigma^2 = \left[ \frac{V_{\text{max}}}{N_{\text{max}}} \right]^2 N + \left[ \frac{V_{\text{max}}}{D} \right]^2 
\]

(10-14)

Using Eq. 10-14 the signal to noise ratio (SNR) at 100% contrast for a digital image is

\[
SNR = \frac{V}{\sigma} = \frac{V}{\sqrt{\frac{V_{\text{max}}^2}{N_{\text{max}}^2} N + \frac{V_{\text{max}}^2}{D^2}}} = \frac{V}{\frac{V_{\text{max}}}{\sqrt{\frac{N_{\text{max}}^2}{N_{\text{max}}^2} + \frac{1}{D^2}}}} 
\]

(10-15)

Since the analog signal is proportional to the number of photons \( (N) \) that we associate with a pixel area in the digital image

\[
\frac{V}{V_{\text{max}}} = \frac{N}{N_{\text{max}}} 
\]

(10-16)

we can express the analog signal to noise ratio in terms of the number of photons (per pixel) \( N \) at the input phosphor of the image intensifier (if 100% are absorbed) as

\[
SNR = \frac{N}{\sqrt{\frac{N_{\text{max}}^2}{N_{\text{max}}^2} + \frac{1}{D^2}}} = \sqrt{\frac{N}{N_{\text{max}}^2} + \frac{1}{D^2}} 
\]

(10-17)

We now will investigate this relationship in several important special cases.

**Case 1: High Dynamic Range Systems**

(Ideal system)

If the system dynamic range \( D \) is large compared to \( N \), then the system signal-to-noise ratio (equation 10-17) reduces to

\[
SNR = \sqrt{N} 
\]

(10-18)

in which case electronic noise is negligible and the
DQE of the system is unity. Thus, when the dynamic range is very large, the noise is contributed entirely by photon statistics and is Poisson distributed.

**Case 2:** High signal levels ($N = N_{\text{max}}$)

If we are operating at a high signal level where the number of photons used to generate the image is near the maximum value for the system, then

$$SNR = \frac{N_{\text{max}}}{\sqrt{N_{\text{max}}^2 + \frac{N_{\text{max}}^2}{D^2}}} = \frac{1}{\sqrt{1 + \frac{1}{D^2}}}$$  \hspace{1cm} (10-19)

For example assume an exposure of $X = 16$ mR, a 0.5 mm x 0.5 mm pixel area, $E= 60$ keV, and a dynamic range of $D = 1000$. At 60 keV, the mass energy absorption coefficient of air is

$$\left(\frac{\mu_e}{\rho}\right)_{\text{air}} = 0.0289 \text{cm}^2/\text{gm} \hspace{1cm} (10-20)$$

so that the photon fluence at 60 keV corresponding to an exposure of $X = 16$ mR (from Eq 9-21) is

$$\Phi = 5.03 \times 10^8 \frac{\text{photons}}{\text{cm}^2} \hspace{1cm} (10-21)$$

Over a 0.5 mm x 0.5 mm pixel area, the number of photons is

$$N_{\text{max}} = (5.03 \times 10^8 \text{ photons/cm}^2) (0.05 \text{cm})^2 = 1.26 \times 10^6 \text{ photons} \hspace{1cm} (10-22)$$

Therefore the system signal-to-noise ratio (i.e. the ratio of the maximum signal $V_{\text{max}}$ divided by the standard deviation of the system) is

$$SNR = \frac{1}{\sqrt{\frac{1}{N_{\text{max}}} + \frac{1}{D^2}}} = \frac{1}{\sqrt{\frac{1}{1.26 \times 10^6} + \frac{1}{1 \times 10^6}}} = 747 \hspace{1cm} (10-23)$$

At this point we ask whether this level of signal-to-noise is adequate to see a small low-contrast object in a noisy image. This question can be answered approximately by using the Rose model which relates SNR, contrast, size and fluence. The Rose model states that

$$k^2 = C^2 \Phi A = C^2 N \hspace{1cm} (10-24)$$

where $k = 5$ (a constant specifying the SNR at which the low contrast signal is visible) and
\[ N = \Phi \ A = \text{number of photons used to image object of area } A \]
\[ C = \text{contrast level of signal} = \Delta N/N \]
\[ \psi = \text{photon fluence in background region} = \frac{N}{A} \]
\[ A = \text{area of object} \]
\[ \Delta N = \text{difference in number of photons used to image the object and a background region of equal area.} \]

Therefore, from the Rose model

\[ k^2 N = C^2 N^2 = \left( \frac{\Delta N}{N} \right)^2 N^2 = (\Delta N)^2 \]  \hspace{1cm} (10-25)

Let \( N = \sigma^2 \) be the variance of the noise. Therefore, an alternative way to express the Rose model is to state that

\[ k \sigma = \Delta N \]  \hspace{1cm} (10-26)

Intuitively, this states that the difference in the background and object’s signals must be \( k \) times the standard deviation of the noise. From our derived value of the signal-to-noise ratio at 100% contrast,

\[ \text{SNR} = \frac{N}{\sigma} \]  \hspace{1cm} (10-27)

and rearranging to follow the format of (10-26) we have

\[ \text{SNR} \sigma = N \]  \hspace{1cm} (10-28)

Now dividing equation 10-26 by equation 10-28 and substituting values for \( k \) from the Rose Model equation and SNR from equation 10-23 yields

\[ C = \frac{\Delta N}{N} = \frac{k}{\text{SNR}} = 5/747 = 0.67 \% \]  \hspace{1cm} (10-29)

Thus the operating level where the image intensifier is irradiated at its maximum exposure level, the contrast of a minimally perceptible target is

\[ C = \frac{\Delta N}{N} \approx 1\% \]  \hspace{1cm} (10-30)

Note that this low contrast detectability limit is determined as the ratio of SNR (or \( k \)) from the rose model to the SNR produced by the imaging system at 100% contrast.

**Case 3:**

If \( N = 1/10 \ N_{\text{max}} \), then
\[ SNR = \frac{\frac{N}{\sqrt{N + \frac{N_{\max}^2}{D^2}}} \frac{10}{\sqrt{\frac{N_{\max}}{10} + \frac{N_{\max}^2}{D^2}}}} \]

With \( N_{\max} = 1.26 \times 10^6 \) photons and \( D = 1000 \) as before, we have \( SNR = 96.3 \). From equation 10-29, when \( SNR = 96.3 \), an observer would be able to see a minimal contrast level of

\[ C = \frac{\Delta N}{N} = \frac{k}{SNR} = \frac{5}{96.3} = 5.2\% \]

Case 4:

If \( N = \frac{1}{100} N_{\max} \), then

\[ SNR = \frac{\frac{N}{\sqrt{N + \frac{N_{\max}^2}{D^2}}} \frac{10}{\sqrt{\frac{N_{\max}}{10} + \frac{N_{\max}^2}{D^2}}}} \]

With \( N_{\max} = 1.26 \times 10^6 \) photons and \( D = 1000 \), we have \( SNR = 9.96 \) in which case only contrast levels greater than

\[ C = \frac{\Delta N}{N} = \frac{k}{SNR} = \frac{5}{9.96} = 50.2\% \]

are visible in the image. At this point, the SNR of the image is severely limited by the electronic noise, not by quantum statistics.
10.6 Methods to Improve the Noise Characteristics of Digitally Subtracted Radiographs

10.6.1 Removal of Bright Spots

From the above examples, it is obvious that digital radiographs obtained with image intensifier systems will be severely limited by electronic noise if the image is not obtained with the maximum number of photons $N_{\text{max}}$. What is not obvious is that we often are forced into this undesirable situation since the image contains regions with both high and low x-ray transmission. High transmission is seen at the edge of the patient or in body regions containing air (lungs or bowel gas). When these regions of high x-ray transmission are imaged at maximum video levels, patient regions with lower transmission are imaged at lower video levels where the data are compromised by electronic noise. A common technique to reduce this problem is to place bags of saline over the high transmission areas “bright areas” or to place pieces of aluminum in the x-ray beam to decrease the exposure to regions where the patient is transmissive (Figure 10-13). Ideally, the input exposure field can be tailored to deliver a nonuniform exposure field to the patient so that the exposure field reaching the image intensifier is uniform. In this case, all regions of the patient are imaged closer to maximum exposure levels, to obtain the highest possible signal-to-noise levels in all areas of the image.

10.6.2 Role of the Video Camera Aperture

Another method to improve the signal-to-noise ratio in digital subtraction angiography is to increase the exposure delivered to the patient, decreasing the noise contribution from quantum statistical sources. However, because a specific light level delivered to the camera target will produce a maximum video response, the x-ray exposure cannot be increased indefinitely without making other adjustments in the system to insure that this maximum light level is not exceeded. The video camera aperture has a fundamental role in this respect to control the level of quantum noise in the digitally subtracted angiogram. Because the aperture is located between the output phosphor of the image intensifier and the input optics of the video camera, decreasing the aperture diameter also decreases the amount of light reaching the camera target and lowers the camera response for a given x-ray exposure level. Correspondingly, the x-ray exposure level must be increased while the aperture diameter is decreased to maintain a constant video signal level. When the camera aperture is decreased, more x-ray photons are used to acquire the image at the quantum sink (i.e. the input phosphor of the image intensifier). Therefore, the overall signal-to-noise ratio of the video signal increases (assuming that the x-ray
exposure is adjusted to maintain a maximal video signal in the patient image). This reduces the degree of quantum statistical noise and improves the overall noise characteristics of the image.

It is important to stress the complementary roles of the x-ray exposure level and the video camera aperture. If the x-ray exposure level is increased without adjusting the aperture, then the increased light output of the image intensifier can drive the video camera into saturation, producing a useless signal that saturates at its maximum level. Similarly, decreasing the camera aperture, while holding x-ray exposure fixed will reduce the light delivered to the video camera, resulting in a smaller video signal. The quantum noise component scales with the video signal, but the SNR of the video signal will be reduced due to the fixed level of electronic noise in the video system. Thus, the camera aperture must be adjusted to provide a video signal near the maximum level to avoid electronic noise, and this signal should correspond to the region of interest in the body.

10.6.3 Image Integration

A final way to improve system SNR is to use various processing schemes that add (or "integrate") images together either before or after subtraction to average out the noise contained in the digital radiographic images. The simplest way this can be performed is by means of frame integration where two or more frames of the image are added together, in an attempt to reduce both quantum statistical and electronic noise contributions in the final digital radiograph. If \( M \) frames are added together, where all the frames are nearly identical except for their random noise content, and if \( \sigma \) represents the noise in each image, then the noise in the integrated image increases by \( M^{1/2} \sigma \) while the signal increases by \( M \). Therefore, the SNR improves by \( M^{1/2} \). Frame integration has the advantage of reducing both the effects of photon statistical as well as electronic noise sources. By comparison increasing radiation exposure per frame only improves on quantum noise effects. However, frame integration has the serious disadvantage that it is more prone to motion artifacts since a longer period of time is used to acquire a single integrated image.

10.7 Spatial Resolution in Digital Subtraction Angiography

There are several factors to consider concerning spatial resolution in digital subtraction angiography. The first is the digital matrix format (512x512 or 1024x1024) used to acquire the image data. The second is the spatial resolution of the image intensifier. The third is the degree of geometric unsharpness due to focal spot size. As we discussed in previous lectures, there is a fundamental trade-off between the increase in object detail that can be seen due to image magnification and the loss in object detail due to increased geometric unsharpness. The rendition of object detail increases with greater magnification due to the fixed resolution in the image intensifier and digital image matrix. On the other hand, increasing geometric unsharpness (magnification of focal spot) degrades spatial resolution in the object with greater object magnification.

If an image is recorded with an object magnification of \( M \) and if the focal spot has width \( "a" \), the width of the focal spot when projected onto the detector is \((M-1)a\). This width is equal to that of a structure in the object plane having a width of \( \frac{(M-1)a}{M} \). We therefore can characterize the cut-off frequency \( u_s \) (the frequency suppressed by unsharpness) of focal spot blurring (i.e. geometric unsharpness) by the inverse of the resolution width.
Similarly, the resolution width of $d$ for the image intensifier is $\frac{d}{M}$ in the object plane, giving a cut-off frequency from the image intensifier of

$$u_n = \frac{M}{d} \quad (10-36)$$

Finally, if the detector width is $D$ and the image is digitized into an $N \times N$ matrix, the Nyquist frequency imposes a limit on the spatial frequencies that can be faithfully reproduced in the image. For an object magnification of $M$, the limiting spatial frequency $u_{\text{dig}}$ imposed by the digitization process is:

$$u_{\text{dig}} = \frac{N}{2} \frac{D}{M} \quad (10-37)$$

The trade-off between the decrease in detector unsharpness and the increase in geometric unsharpness with increasing object magnification is best seen by graphing the cut-off frequencies for focal-spot blurring and for detector response as a function of object magnification. As shown in Figure 10.14, the curve labeled "source" shows how increasing magnification produces the resolution loss (i.e. decreasing the cut-off spatial frequency) due to geometric unsharpness.

A focal spot size of approximately $a = 1$ mm is common in digital subtraction angiography, corresponding to a cut-off frequency of

$$u_s = \frac{M}{(M-1)a} \quad (10-35)$$

We will consider two different components of detector resolution, from the image intensifier and from the digital image matrix. The image intensifier (II) has a spatial resolution of about $d = 0.2$ mm giving a cut-off frequency of $\frac{M}{d}$.

If we assume that a 512 x 512 image matrix is used to digitize an inscribed circular field of view with a diameter of 23 cm (approximately 9 inches), then the pixel width is

$$\frac{N}{2} \frac{D}{M}$$

This yields a cut-off spatial frequency of

$$u_{\text{dig}} = \frac{MN}{2D}$$

Figure 10.14. The spatial resolution of a DSA image can be limited by geometric unsharpness from the x-ray focal spot, by image intensifier resolution, or by aliasing from the digital image matrix. The limitations of image intensifier resolution and aliasing decrease, while that from geometric unsharpness increases with magnification.
\[ u_{\text{dig}} = 1.11 \text{ M mm}^{-1} \quad (10-41) \]

In this example the digital image matrix rather than the image intensifier limits the spatial resolution of the detector at low magnification. However, at higher magnification, geometric unsharpness becomes more problematic. The intersection between the cut-off spatial frequency curve for the digital image matrix and that for focal spot blurring indicates the magnification at which the resolution of the imaging system is optimum.
CHAPTER 10: HOMEWORK PROBLEMS

1. You are visiting a hospital that has just acquired a digital subtraction angiography (DSA) system. You watch as the radiologist sets the kVp and mA to obtain an exposure level that gives a maximum video signal output. The radiologist then looks at the settings on the x-ray console and decides to decrease the patient exposure and tolerate the accompanying increase in noise by decreasing the x-ray tube current (mA) to 25% of its original value. You immediately remember that it is important to adjust the video camera aperture for this new exposure level and alert the radiologist of the problem. The radiologist thanks you for the information, readjusts the aperture to achieve a maximum video signal output with the lower exposure level, and then continues the examination.

For the following calculations assume that the dynamic range of the video camera is 1000:1. Before the radiologist decreased the exposure or increased the camera aperture, a maximum video signal was obtained at a setting of 70 kVp and 200 mA. At this setting for a 40-msec exposure, the x-ray tube produces a photon fluence of \(1.031 \times 10^{6}\) photons over a resolution area of 1 mm\(^2\).

a. When the initial exposure settings were established, the diameter of the aperture was 4 mm, corresponding to an f-stop of f/5.6. What is the correct diameter and f-stop of the camera aperture after the patient exposure has been decreased to 25% of its original value?

b. How is the signal-to-noise ratio of the image affected if the patient exposure is reduced by a factor of 4 but the camera aperture is not adjusted for the new exposure level?

c. What is the signal-to-noise ratio of the image if the patient exposure is reduced by a factor of 4 and the camera aperture is adjusted to maintain a maximal video signal output?

d. Explain to the radiologist why the video camera aperture must be adjusted for the new exposure level. Use your calculations from parts b and c of this problem to guide your thought processes, but give the radiologist conceptual and intuitive (rather than quantitative) explanations.

2. You take a research position in an x-ray imaging laboratory at a major American university. Your first development is a new video camera especially developed for pediatric coronary angiography. Because the infant heart beats at a higher rate than that of the adult, the camera has a video frame rate of 100 frames per second. Second, because you want to limit both the radiation exposure and the amount of contrast media required for the angiographic study, the camera is designed with a dynamic range of 4000 to 1. Finally, because the infant is small, the camera is designed with a video signal bandwidth high enough to be compatible with a 2048 x 2048 digital image matrix. You then begin to develop a digital image processor that is compatible with this new video camera.

All of a sudden, your grandmother drops by with a batch of chocolate chip cookies. Grandma has been very lonely lately since Grandpa bought a new Macintosh computer and discovered
"Internet browsing" so she starts asking lots and lots of questions. While munching on her delicious cookies, you try to explain to Grandma what you are doing.

a. Grandma first asks you what a “video signal bandwidth” is, and why a higher “video signal bandwidth” is needed if you want to obtain a 2048 x 2048 (rather than a 512 x 512) image. What problems will you encounter if you have too low of a bandwidth? What problems will you encounter if your camera has too high of a bandwidth? (By the way, if you use the word “aliasing” in her presence, Grandma will make you wash your mouth out with soap.)

b. Grandma then asks you what a "video dynamic range" is and why a higher "video dynamic range" might let you decrease the radiation exposure and amount of contrast media needed for the study.

As you are explaining these things to Grandma, a doctor who is walking through the corridor outside the door of your laboratory distracts her. Feeling rather frisky because of the lack of attention by Grandpa, Grandma decides to chase after this new target rather than listen to you talk about medical imaging. As she runs out the door, she almost knocks over Bruce Hasegawa who overheard your conversation, lost interest in an nurse he was talking to in the hall, and has entered the room to join the conversation. Noting the expression of intense curiosity on his face, you moan, wishing Grandma were asking questions rather than Bruce. By now, Grandma has left you alone with Bruce who, to your horror, is busily gobbling up the cookies and who also is starting to ask some questions about your new invention.

c. How many bits are needed by the analog-to-digital converter to minimize quantization errors in the digitized image data?

d. If you eventually want a 2048x2048 image at a rate of 100 video frames per second, what is the digitization rate of the analog-to-digital converter for a digital image processor compatible with this video camera? What is the maximum video bandwidth the camera should deliver to be consistent with the sampling rate of the analog-to-digital converter?

e. The image intensifier has two different operating modes, one with a 6-inch diameter field-of-view and other with a 9-inch diameter field-of-view. What spatial frequency (in cycles/mm), as measured in the detector plane, will the 2048 x 2048 image matrix support for each of the two operating modes?

f. Assume that your image intensifier has a point-spread function width of 200 microns and that you operate the image intensifier with a 6-inch diameter field of view with an x-ray tube having a 300 micron focal spot size. Determine the optimal magnification for imaging the infant heart and the corresponding optimal cut-off spatial frequency for this system? Is this magnification compatible for imaging a 2-inch diameter infant heart and sufficient to see 400 micron diameter coronary arteries?

g. The infant thickness is equivalent to 15 cm of water and the effective energy of the x-ray beam is 30 keV. What should the entrance exposure to the infant be if you want to see a
1% contrast level in a 400 micron diameter infant coronary artery with your video camera?

h. Bruce comments that your calculations for parts c and d of this question show that you need an analog-to-digital converter which is technically impossible to design and build at this time. However, he tells you that the important issue in coronary imaging is one of limiting motion unsharpness rather than one of frame rate. He would rather have a few images each with minimal amounts of motion unsharpness rather than lots of images acquired over the entire cardiac cycle. In other words, he prefers an imaging system with a short acquisition time per frame, but does not need one with a video frame rate faster than about 15 frames per second.

With this in mind, you start thinking about using a video camera with a progressive readout to be used in combination with a short x-ray exposure time. If you want to maintain the optimal spatial resolution as defined by your calculation in part f of this question, how short of an exposure time do you need if the infant's myocardium has a maximum velocity of 5 cm/sec at the point of maximum cardiac contraction.

3. Let's think a little about digital look-up tables such as those utilized in digital subtraction angiography for the logarithmic transformation. (We can offer you a hint that a digital look-up table is built out of random access memory).

a. How might you design a digital look-up table in hardware? How would you load the digital look-up tables with the transformed values? What circuit design would allow you to deliver digital values and obtain the transformed values from the digital look-up table? How much random access memory (RAM) is needed for a look-up table with a 10-bit input and a 10-bit output?

b. If the look-up table has a 10-bit input, the largest value which will be delivered to the look-up table will be 1023 (i.e. $2^{10} - 1$). However, the natural logarithm of 1023 is 6.93. Describe how we can represent the logarithmically transformed values if we want a 10-bit output from the look-up table. You should be able to give a specific mathematical algorithm to make this possible. What effect will this have on the digitally subtracted angiograms?

4. Compare the contrast resolution (at peak video signal) associated with a TV fluoroscopy system with a dynamic range of 1000:1 for the following cases. Assume that the dynamic range refers to a characteristic resolution element of (0.5 mm$^2$).

a. Fluoroscopy at an exposure of 1 mR per image.

b. Digital radiography at an exposure of 1 R per image (neglect digitization noise).