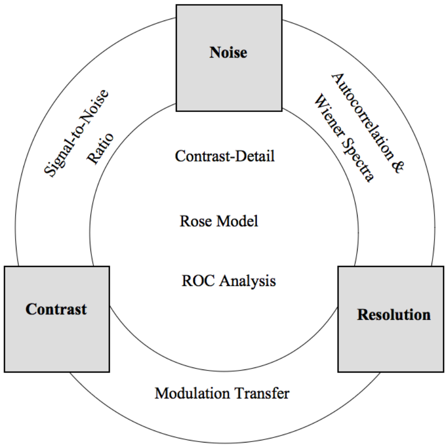
CHAPTER 1: INTRODUCTION TO THE PHYSICS OF MEDICAL IMAGING

This book presumes that you are knowledgeable about the physical mechanisms underlying the formation of the medical x-ray image. You should have a basic understanding of the interaction of electrons with matter (bremsstrahlung) and the formation of x-rays, the interaction of x-rays with matter (photoelectric and Compton interactions), how the medical x-ray image is formed with intensifying screens and radiographic film, how (and in what units) the quantity of radiation is measured, and finally have an appreciation of the principles of radiation protection. There are several excellent texts which cover these topics, including those by Johns and Cunningham, Hendee, and Ter Pogossian, Currey et. al. (*Christensen's Physics of Diagnostic Radiology*), and Bushberg et al., (*The Essential Physics of Medical Imaging*). You are urged to consult these references if the aforementioned topics are not familiar to you.

1.1 Description of the medical image

Figure 1-1 Medical Imaging Concept Overview



We start our discussion with three basic concepts used to describe an image: spatial resolution, contrast, and noise. These fundamental concepts serve as the basis for characterizing imaging and associated instrumentation. We will also discuss broader intermediate concepts that tie together pairs of basic concepts. The intermediate concepts are modulation transfer function (MTF), Wiener spectrum, and signal-to-noise ratio (SNR). Finally, the Rose model (and the related contrast detail curves), and receiver-operator characteristic (ROC) curves provide a means to unify all three basic concepts.

Figure 1-1 illustrates imaging concepts used to model imaging system performance. The most basic concepts of noise, spatial resolution, and contrast are indicated in boxes, while intermediate linking concepts of signal-to-noise ratio, Wiener spectrum, and modulation transfer function are indicated in the areas bridging pairs of basic concepts. Finally, the most comprehensive concepts (Rose Model, contrast detail analysis and ROC analysis) lie at the center since they include elements of all concepts, at least implicitly if not explicitly. This diagram is not intended to suggest a hierarchy, that one imaging concept somehow is more important than another. Rather, the student who ponders Figure 1-1 will appreciate the unity of medical imaging science and will understand these fundamental concepts as part of the whole, rather than as unrelated and independent elements. To help understand this diagram we will define the terms conceptually.

Intuitively, the spatial resolution of an imaging system can be defined in terms of the smallest spacing between two objects that can be imaged clearly. For example, the spatial resolution of a conventional x-ray system with direct film exposure is approximately 0.01 mm while that of a CT scanner is approximately 1 millimeter. A satellite orbiting the earth's surface can record an object that is a few feet in size. In each case, the distance between separate objects that a device can record is one measure of its spatial resolution. This conceptual definition of spatial resolution is used widely in medical imaging. Another more quantitative definition specifies the resolution of an imaging system in terms of its "point spread function", which is the image of an "ideal" point object.

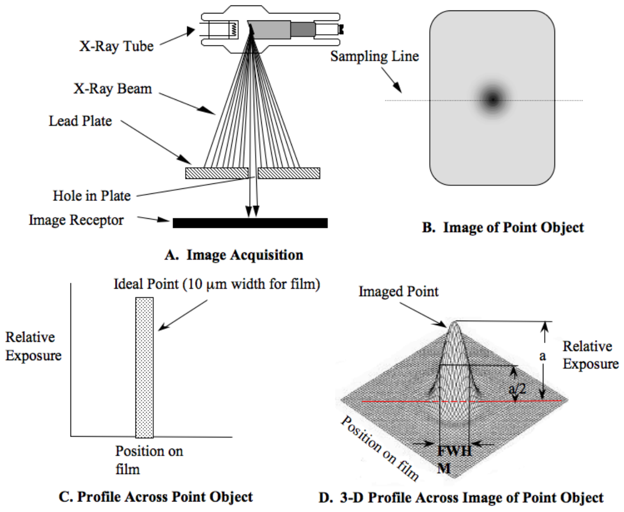


Figure 1-2. Spatial Resolution

Figure 1-2 illustrates how spatial resolution of an imaging system might be measured. In this example, the x-ray system images a "point" object, which is a small hole in an otherwise radio-opaque sheet of lead. X-rays from the source pass through the small hole, forming an image of the point object. We can determine the point spread function (PSF(x,y)) of the system by recording the optical density (OD) values across the point image using a scanning optical microdensitometer if the image is recorded on a sheet of film, or by extracting digital values if the image is recorded digitally. The graph of the measured values (OD or other), when converted to relative exposure, as a function of position is the point spread function. A 1-D profile through the center of a 2-D PSF is shown in Figure 1-2D.

Ideally, one would see an exact representation of the object in the image, in that the width of the image would exactly match the width of the object. However, the image of the point object is always blurred by the imaging system. The amount of blurring by an imaging system can be characterized in terms of the full-width-at-half-maximum (FWHM) of the point spread function (Figure 1-2D). In Chapter 6 we discuss how the point spread function of an imaging system can be determined experimentally, and describe other ways to characterize a system's spatial resolution.

Image Contrast is a measure of difference between adjacent regions in an image. In medical images contrast is commonly used to assess difference between adjacent tissues. For x-ray imaging, contrast between bone and soft tissue is high and contrast between fat and muscle is low. Radiographic contrast depends on several different factors including the chemical composition of the object, the type of device used to record the image (whether it is film or an electronic detector), the energy spectrum of the x-ray beam, whether or not scatter radiation is present in the x-ray beam, and whether fog or some other baseline signal is present in the imaging device.

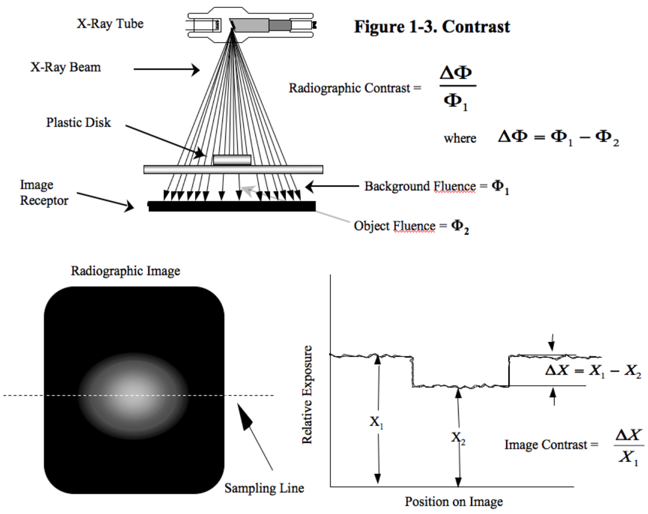


Figure 1-3. Contrast

Contrast is normally expressed as relative contrast and this concept is illustrated in Figure 1.3. In this example, a small plastic disk is radiographed. The background material (e.g., plastic platform) attenuates the x-rays passing through the background, while both the disk and the background material attenuate x-rays within the disk region. Thus the x-ray fluence (photons/area) is higher outside of the plastic disk than it is beneath it. The film density of the processed x-ray film is approximately proportional to the logarithm of the x-ray fluence. The light intensity transmitted through the illuminated x-ray film is lower in the background than beneath the disk. If the transmitted light intensity for the background is I_1 and that for the disk is I_2, then the contrast of the disk relative to the background is defined as:

$$C = (I_2 - I_1)/I_1 \tag{1.1}$$

Contrast is unit less and can be positive or negative, though we usually ignore the sign since it is understood from the context of the measurement. In this example image contrast was positive; however, radiographic or exposure contrast was negative. This contrast reversal is a property of x-ray films that produce negative images. We will investigate factors contributing to image contrast more thoroughly in Chapter 4.

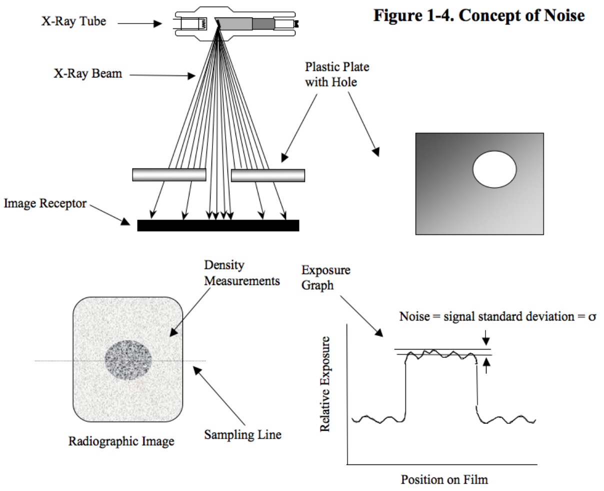


Figure 1-4. Concept of Noise

Random Noise relates to the uncertainty or the imprecision with which a signal is recorded. In impressionist paintings, the artist often would create an image by painting a large number of small dots on the canvas. There is a great deal of uncertainty in the image being created when a small number of dots have been placed on the canvas. As the number of dots increases, the precision or certainty with which the image is being represented increases. A similar thing happens in x-ray and radionuclide imaging. An image that is recorded with a small number of photons generally has a high degree of uncertainty or is very noisy, while an image recorded with a large number of photons is very precise or is not noisy. Grains in the radiographic film, grains in the intensifying screen, or electronic noise that is present in an electronic circuit or electronic detector can also contribute to system noise. In all cases these factors contribute to the uncertainty or imprecision with which a signal is recorded, i.e. to random noise.

An example of how noise might be measured is given in Figure 1-4. Here an image is acquired of a sizeable plastic plate with a hole in it. The plate is placed on a similar plastic platform. A scanning microdensitometer is used to measure the density across the exposed radiograph. Ideally, we could repeat this experiment and obtain exactly the same densitometer readings each time. However, it is seen that due to the random nature of emission of x-rays, attenuation within the plastic plates, and exposure of the film, this does not occur. Therefore, each density recorded is done with some level of uncertainty. This uncertainty is referred to as random noise. While there are several ways to quantify noise, the most simple is to determine the standard deviation of the image values about their mean in a uniform region of an image. The concept of noise, how it is quantified, and its impact on images will be discussed in greater detail in Chapter 8.

Imaging science would be simple if an imaging technique could be described, and its performance quantified, in terms of only one of these factors: noise, spatial resolution, or contrast. In such a simple world, one could say that a screen-film combination has better spatial resolution than a CT scanner and therefore is better for all medical imaging applications, that a CT scanner is better than a film-screen system because it provides higher contrast, or that MRI is better than CT because of its high soft tissue contrast. These simple statements obviously are not true in all cases and each imaging modality is better suited to specific medical imaging needs. To evaluate the suitability of a specific system for a given imaging task, scientists must understand the tradeoffs in the design of the instrumentation used to image the human body as well as the needs of the imaging task.

For this reason several intermediate linking concepts are used to tie together the simple descriptors of noise, spatial resolution, and contrast. It is insufficient to specify the resolution of an instrument only in terms of the size of an object that it can image. In fact, the contrast with which an object is represented decreases as the object size approaches the FWHM of the system point spread function (even though its thickness is unchanged). An intermediate concept that is often used to help clarify this is the modulation transfer function or MTF(f). The MTF describes the contrast produced by an imaging system as a function of the spatial frequency of the object. The MTF(f) is readily described in terms of periodic objects that appear sinusoidal to the imaging system as in Figure 1-5. The measurement of MTF(f) will be covered in Chapter 6.

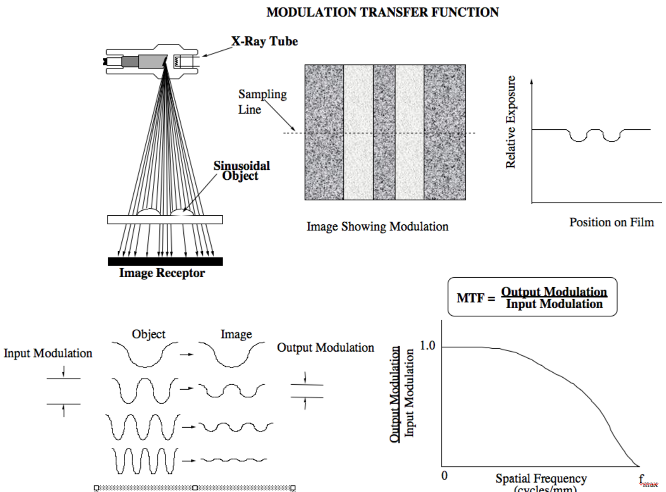


Figure 1-5: MTF Concept

Another important factor relating to how well an observer can see an object is the signal-to-noise ratio, certainly a familiar term to electronic engineers or scientists working with low-level signals in electronic circuits. As the name implies, the signal-to-noise ratio (SNR) is the ratio obtained when the signal in an image is divided by the noise in the image (both measured in same units). Typically the signal is defined as the difference between an object and its background (numerator of contrast equation), and the noise is determined as the uncertainty with which that object is recorded (usually the standard deviation). Conceptually, if one were imaging a tumor that was found in the liver, the signal would be the difference between the tumor and the surrounding material while the noise would be the standard deviation of the signal level within the tumor. The ratio of these two numbers would be the signal-to-noise ratio.

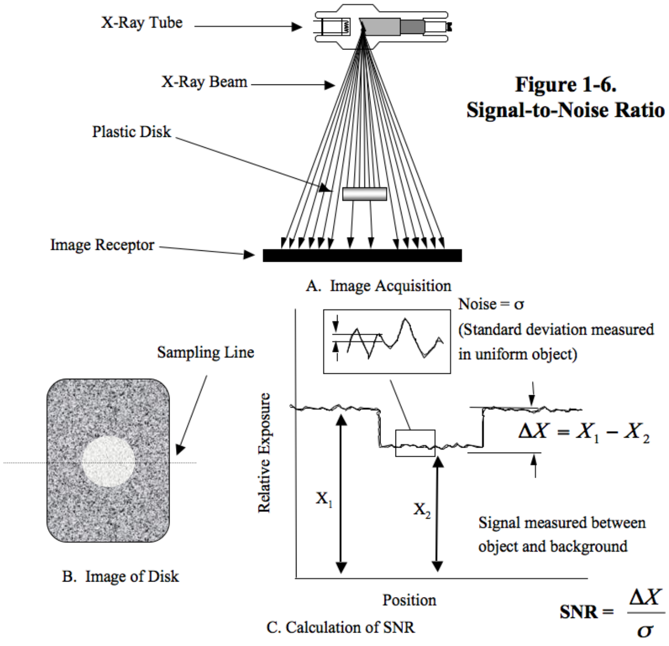


Figure 1-6. Signal-to-Noise Ratio

The third and final intermediate linking descriptor concept is known as the <u>Wiener spectra</u>. It represents the level of noise as a function of the spatial frequency and is similar to the MTF. Just as the ability of an imaging system to record a image contrast decreases as the object's spatial dimension becomes smaller and smaller, the ability of the system to record noise fluctuations decreases as the fluctuations become smaller and smaller in their spatial extent. The Wiener spectrum describes the noise amplitude as a function of spatial frequency and equals the Fourier Transform of the autocorrelation function in a uniformly exposed radiographic image (Figure 1-7). This topic will be explored further in Chapter 8.

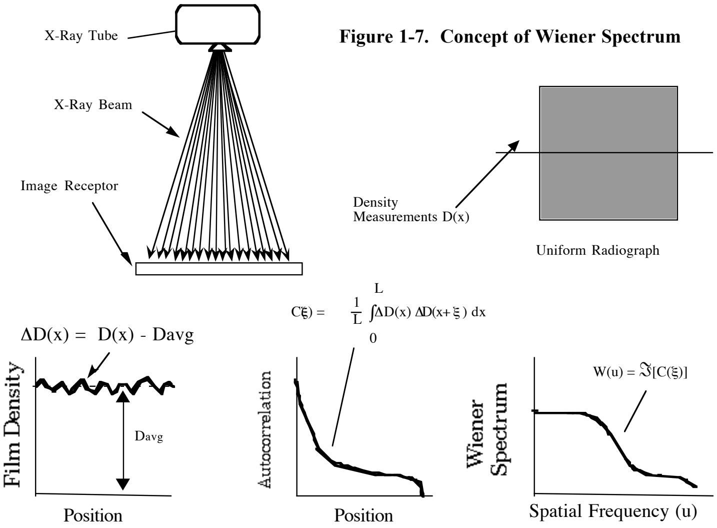


Figure 1-7. Concept of Wiener Spectrum

Finally, there are image quality concepts that attempt to include the three basic concepts of spatial resolution, contrast, and noise, as well as the performance of the observer, in the evaluation of an imaging system. These methods and descriptions are called the Rose model, the related concept of contrast-detail analysis, and receiver operating characteristic (ROC) analysis.

The <u>Rose model</u> provides a simple mathematical equation (1-2) for the relationship between SNR (k), object size (A), and contrast (C) as follows:

$$k^2 = C^2 N = C^2 \Phi A \qquad (1\text{-}2)$$

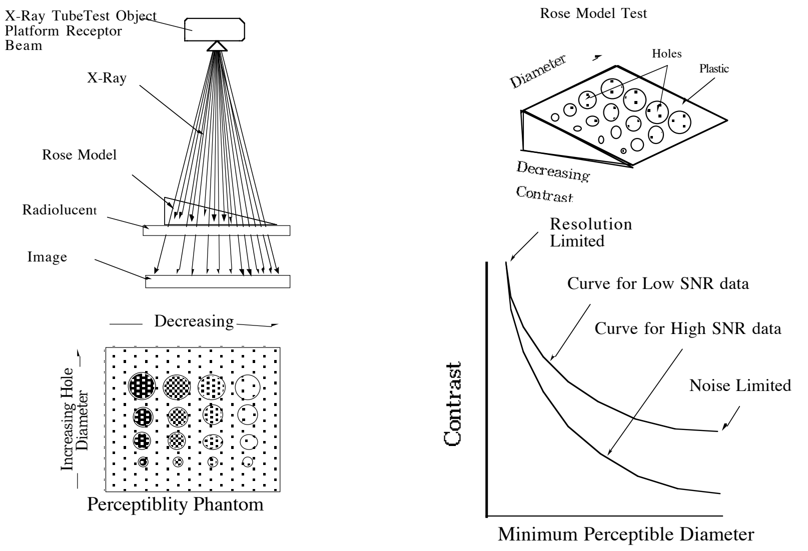
where

k = SNR needed to just see an object in an image
C = contrast of the object with respect to surrounding background
N = number of photons used to image the object of area A
A = area of the object
Φ = photon fluence (N/A) used to form the image

.

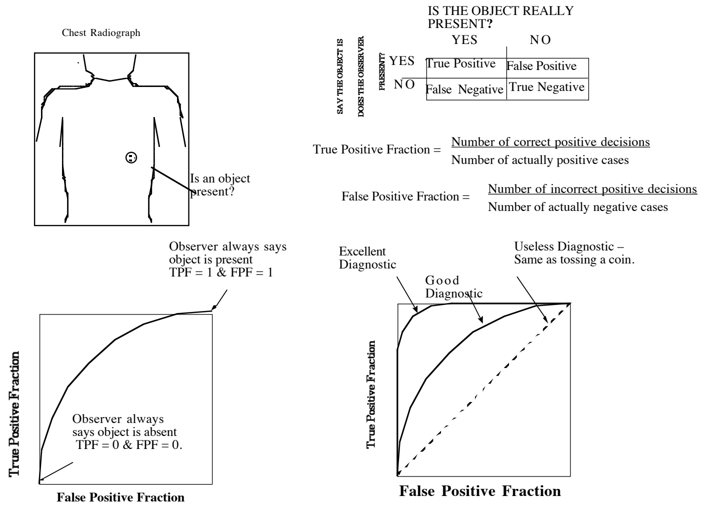
As such the Rose model relates the three fundamental concepts of spatial resolution (i.e. object size), noise, and contrast and predicts whether an object with a given set of characteristics (size and contrast) can be visually detected in an image created at a certain noise level. The value of k (SNR) in the range of 5 to 7 has been reported to be adequate for many imaging tasks. With a fixed value of k we can estimate the size of the smallest object (A) we might be able to see at contrast level (C) with photon fluence (Φ) using equation (1-2). The Rose model is a key element in our estimation of the observability of low contrast objects in a noisy image, conditions often found in radiology. We will describe the Rose model in much greater detail in Chapter 9.

Figure 1-8. Contrast-Detail Curves



In the related concept of <u>contrast detail analysis</u>, an observer reports the size of the smallest object he or she can perceive at a certain contrast level and with a given noise level in the image. Smaller objects must have higher contrast to be seen in the image. The result of such an analysis is a "contrast detail" curve in which the size (i.e. detail) of smallest observable objects are plotted against their contrast for a given noise level (Figure 1-8). Indeed these curves are graphical representations of the Rose model. Smaller objects must have higher contrast to be seen in the image. An imaging system with low photon fluence (low SNR) would trace out a different curve than one with high photon fluence (high SNR). Such a set of curves, one for each noise level, help describe the relationship between the contrast needed to see a certain sized object (resolution) for a particular imaging system.

Figure 1-9. Receiver operating characteristic (ROC) Curve



Finally, <u>receiver operating characteristic curve (ROC)</u> analysis is considered to be the ultimate test of an imaging system. It generally is used to compare one imaging system against another or one imaging technique against another. In ROC analysis, the <u>true positive fraction</u> of a diagnostic test is plotted against <u>false positive fraction</u>. The true positive fraction is also called the sensitivity of a test, and relates to how well a test performs in detecting a disorder. In other words, if a physician were trying to diagnose the existence of a tumor on a chest film, the true positive fraction would be the fraction of times the physician said there was a tumor (test was positive) when there actually was a tumor in the patient. Similarly, the false positive fraction is fraction of times the physician said there was a tumor when no tumor was present. As trivial examples, if a physician always said that a tumor was present in a chest film then he would call all the examples of tumors correctly in which case his true positive fraction would be 100%. Unfortunately, if he would diagnose all healthy people as having a tumor, and his false positive fraction would be 100%. Alternatively, if a physician always said that no tumor existed his true positive fraction would be 0%, but he also would miss all the tumors on the film so his false positive fraction would be 0%. The receiver operating characteristic curve plots the true positive fraction against the false positive fraction at different levels of confidence. Unlike other performance graphs, ROC graphs do not use noise, contrast, or resolution as dependent or independent variables, but outcomes are dependent on all of these factors. This makes it easier to compare different imaging systems to determine which might be best for a particular imaging task. An ideal system would give no false positives unless the observer insisted upon calling everything positive. Its ROC curve therefore would hug the upper left corner of the graph. On the other hand, if the image conveyed no information at all and the observer was forced to guess whether or not the object was present, the ROC curve would be a diagonal line from the lower left to the upper right corner. In this case, the true positive fraction and the false positive fraction would be equal and the observer would make correct and incorrect decisions about the existence of the disease at the same rate. Therefore, the amount by which the ROC curve bows away from the diagonal and towards the upper left-hand corner is a measure of the usefulness of the imaging technique.

1.2 Organization of Course

In this course, we begin by outlining the basic electronic imaging processes used in diagnostic imaging (Chapter 2), as well as summarizing the basic concepts of electronic imaging (Chapter 3). The following sections each address the topics of contrast (Chapter 4), spatial resolution (Chapter 6), noise, the detective quantum efficiency and signal-to-noise ratio (Chapter 8), and the Rose Model (Chapter 9). Chapter 5 discusses linear systems analysis and the concept of the modulation transfer function, while Chapter 7 discusses statistical processes as a prelude to the chapter on noise. Finally, Chapters 10 and 11 discusses the physics of digital subtraction angiography (DSA) and temporal filtration techniques. A CT Chapter dealing with x-ray CT, SPECT, and PET will be inserted between Chapters 6 & 7. A section on Autocorrelation Functions will be inserted between Chapters 8 & 9. Examples from Nuclear Medicine and Magnetic Resonance imaging will be added where helpful.

Homework Problems

1. Assume that we've taken a radiograph of a plastic coin from which we have made the following brightness measurements along a line including the coin's image:

   359, 376, 421, 424, 394, 371, 423, 349, 399, 346, 482, 476, 498, 501, 528, 449, 501, 530, 525, 439, 502, 467, 521, 520, 523, 479, 528, 529, 476, 523, 430, 392, 439, 390, 429, 439, 387, 380, 420, 429

   (a) Make a graph of these values as a function of position.
   (b) Calculate <u>contrast</u>, <u>noise</u>, and the <u>signal-to-noise ratio</u>.

In each case define any ambiguous terms and describe how you determined each value.

2. Assume that a point spread function is described by the following Gaussian function:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

(a) If f(x) represents a probability density function on the domain (-∞ to +∞□ show that μ is the mean value and σ is the standard deviation.

(b) Show algebraically that the full-width-at-half-maximum (FWHM) for a Gaussian point spread function is:

$$FWHM = 2\sqrt{2\ln(2)}\sigma$$

(c) The following are signal measurements taken at equal intervals across the image of a point object:

Position	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Value	0	1	3	7	14	25	41	61	80	95	100	95	80	61	41	25	14	7	3	1	0

Calculate the standard deviation of the profile, then use the result from (b) to estimate the FWHM of the point spread function.

(d) Graph the signal measurements given in (c) and use this graph to estimate the FWHM. Compare the result with that in (c).

3. Given film density values for a point spread function, explain how you would convert these values to relative x-ray exposure.

4. Conceptually what does the MTF represent?

5. A common method to construct a ROC curve is to have an observer inspect an image and rate their confidence that an object is present, and then check this decision against a "gold standard" (known presence or absence of signal) to determine true positive and false positive rates. Assume that we are using a rating system that ranges from 0 when the observer is the least confident that an object is present through 4 when the observer is the most confident that the object is present.

Assume that an observer has used this rating system to indicate whether or not objects are present in a radiograph. For images where the <u>object actually is present</u>, the observer gives the following scores:

<u>Observer Rating</u>	<u>Number of Decisions</u>
0 – nearly certain not present	2
1 – probably not present	3
2 – uncertain	7
3 – probably present	5
4 - nearly certain present	3

For images where the <u>object is actually not present</u>, the observer gives the following scores:

<u>Observer Rating</u>	<u>Number of Decisions</u>
0 – nearly certain not present	5
1 – probably not present	7
2 – uncertain	8
3 – probably present	3
4 - nearly certain present	2

Generate a ROC for these data.

Hint: The points on the ROC curve are obtained from cumulative ratings.